Definition: 2-tape Turing machine which is similar to a TM but has two tapes, each with its own read/write head. The input is on tape 1 followed by blanks and tape 2 is completely blank. The read/write heads are positioned in the first position. For flexibility, say that the read/write heads can move separately, that is allow stays.

\[ M = (Q, \Sigma, \Gamma_1, \Gamma_2, \delta, q_0, q_{\text{accept}}, q_{\text{reject}}) \]
\[ \delta : Q \times \Gamma_1 \times \Gamma_2 \rightarrow Q \times \Gamma_1 \times \Gamma_2 \times \{L, R, S\} \times \{L, R, S\} \]

A language is Turing-recognizable iff some 2-tape Turing Machine recognizes it.

Proof:
⇒ (only if)
Suppose that a language is Turing-recognizable. Then some Turing Machine recognizes the language. This Turing Machine can be considered a 2-tape Turing Machine which ignores the second tape. Thus, there is some 2-tape Turing Machine which recognizes the language.

⇐ (if)
Plan: Simulate the 2-tape TM on the single tape by placing the significant contents of each tape, separated by new tape symbol #, onto the single tape. By significant contents, use at least one blank for each tape. If the tape has non-blank symbols, keep those on the single tape, along with all interspersed blanks.

Simulate the position of each read/write head on the tape by introducing the new symbols x dot for every \( x \in \Gamma \). x dot indicates that the read/write head is at that location.

For every move, see what is under both read/write heads before deciding what move to make. Perform the appropriate move on the simulated first tape (unmarking the symbol where the read/write head was, and marking the symbol where the read/write head is after the move). Then perform the appropriate move on the simulated 2\textsuperscript{nd} tape.