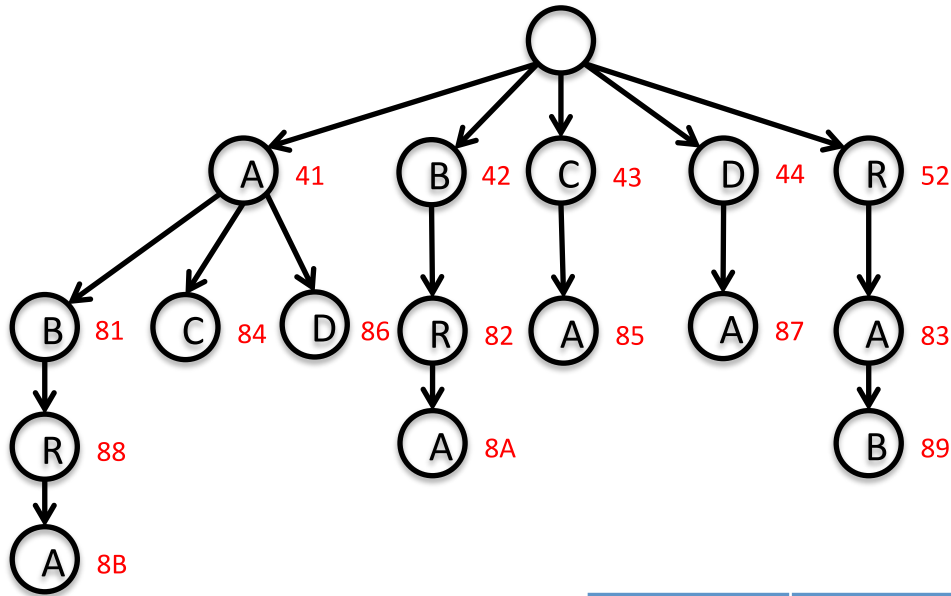
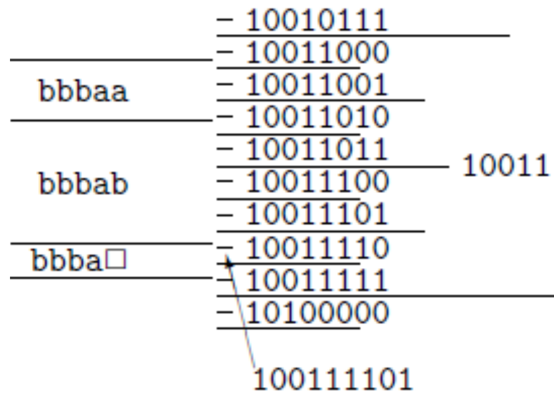
Lossless compression II

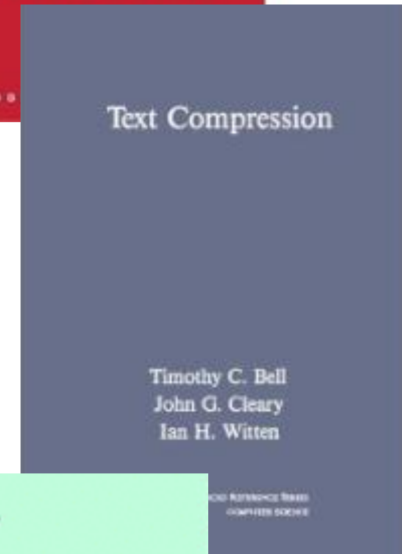


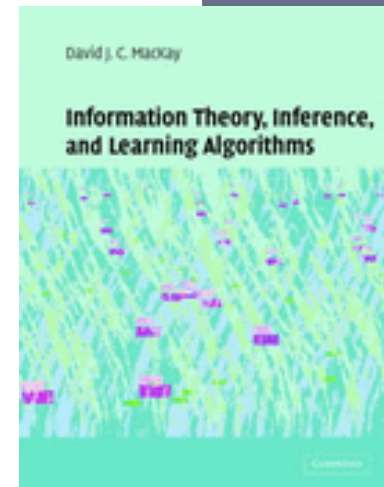


Symbol	Probability	Range
a	0.2	[0.0, 0.2)
e	0.3	[0.2, 0.5)
i	0.1	[0.5, 0.6)
o	0.2	[0.6, 0.8)
u	0.1	[0.8, 0.9)
!	0.1	[0.9, 1.0)

Overview

- Flavors of compression
  - Static vs. Dynamic vs. Adaptive
- Lempel-Ziv-Welch (LZW)
  - Fixed-length codeword
  - Variable-length pattern
- Statistical coding
  - Arithmetic coding
  - Prediction by Partial Match (PPM)





Compression models

- Static
  - Predefined map for all text, e.g. ASCII, Morse Code
  - Not optimal: different texts = different statistics
- Dynamic
  - Generate model based on text
  - Requires initial pass before compression can start
  - Must transmit the model (e.g. Huffman coding)
- Adaptive
  - More accurate modeling = better compression
  - Decoding must start from beginning (e.g. LZW)

LZW

- Lempel-Ziv-Welch compression (LZW)
  - Basis for many popular compression formats
    - e.g. PNG, 7zip, gzip, jar, PDF, compress, pkzip, GIF, TIFF
- Algorithm basics:
  - Maintain a table of fixed-length codewords for variable-length patterns in the input
  - Built progressively by compressor and expander
    - Table does not need to be transmitted!
  - Table is of fixed size
    - Entries for all single characters in alphabet
    - Entries for longer substrings encountered

LZW compression example

- Details:
  - 7-bit ASCII characters, 8-bit codeword
    - For real use, 8-bit → ~12-bit codeword
  - Alphabet: 128 characters + 128 longer strings
  - Codeword: 2-digit hex value
    - 00-79 = single characters, e.g. 41 = A, 52 = R
    - 80 = end of file
    - 81-FF = longer strings, e.g. 81 = AB, 88 = ABR
  - Employs lookahead character to add codewords

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches																	
codeword																	

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword

Mappings found during compression

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A																
codeword	41																

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81

Mappings found during compression

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A	B															
codeword	41	42															

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82

Mappings found during compression

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A	B	R														
codeword	41	42	52														

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83

Mappings found during compression

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A	B	R	A													
codeword	41	42	52	41													

⇧

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84

Mappings found during compression

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A	B	R	A	C	A	D										
codeword	41	42	52	41	43	41	44										

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87

Mappings found during compression

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A	B	R	A	C	A	D	AB									
codeword	41	42	52	41	43	41	44	81									

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87
ABR	88

Mappings found during compression

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A	B	R	A	C	A	D	AB		RA							
codeword	41	42	52	41	43	41	44	81		83							

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87
ABR	88
RAB	89

Mappings found during compression

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A	B	R	A	C	A	D	AB		RA		BR					
codeword	41	42	52	41	43	41	44	81		83		82					

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87
ABR	88
RAB	89
BRA	8A

Mappings found during compression

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A	B	R	A	C	A	D	AB		RA		BR		ABR			
codeword	41	42	52	41	43	41	44	81		83		82		88			

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87
ABR	88
RAB	89
BRA	8A
ABRA	8B

Mappings found during compression

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
matches	A	B	R	A	C	A	D	AB		RA		BR		ABR			A
codeword	41	42	52	41	43	41	44	81		83		82		88			41

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87
ABR	88
RAB	89
BRA	8A
ABRA	8B

Mappings found during compression

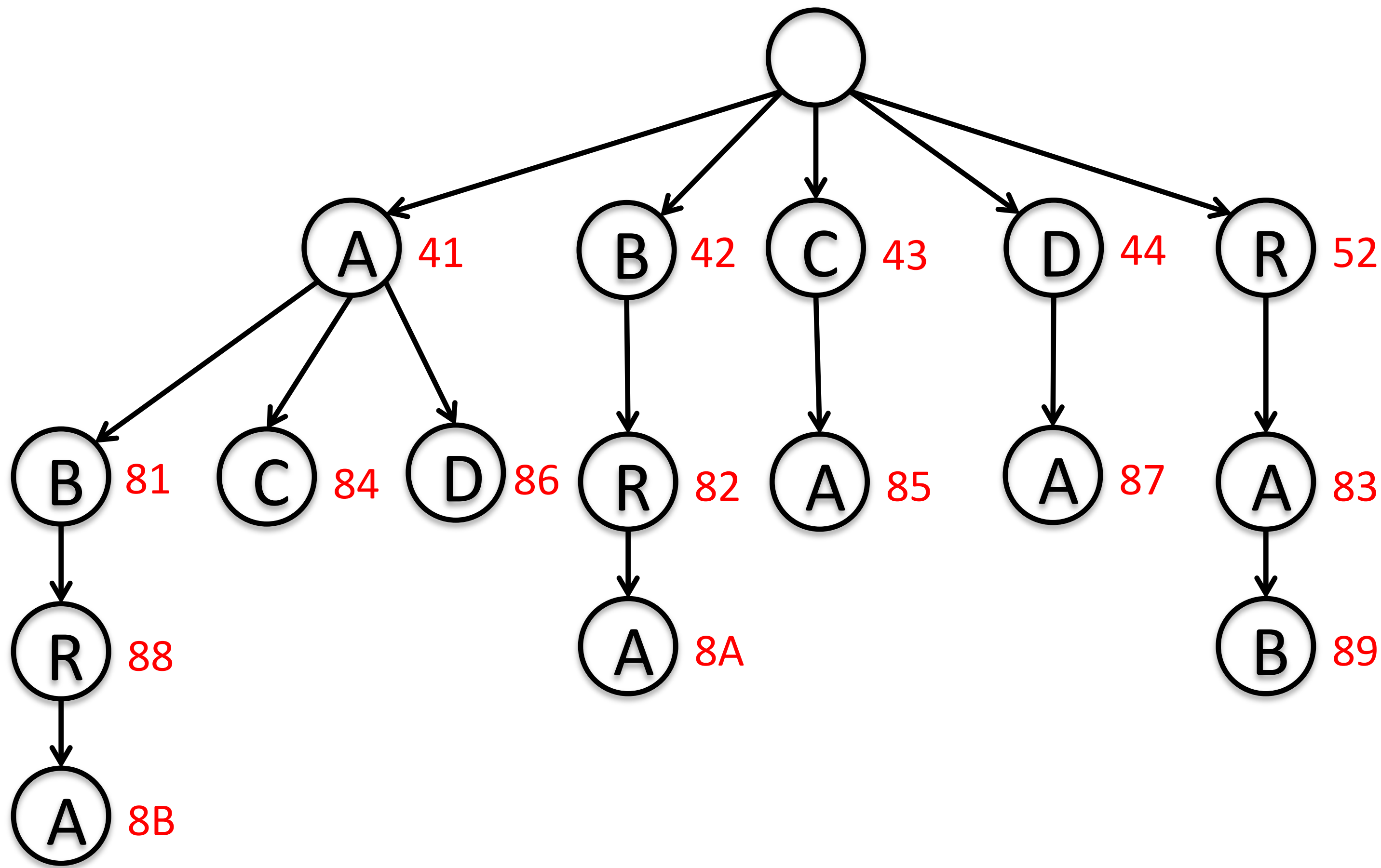
- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

```
Input:
17 ASCII characters * 7 bits = 119 bits
Output:
12 codewords * 8 bits = 96 bits
Compression ratio:
82%
```

Compression data structure

- LZW compression uses two table operations:
  - Find longest-prefix match from current position
  - Add entry with character added to longest match
- Trie data structure, "retrieval"
  - Linked tree structure
  - Path in tree defines string

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c



string	codeword
…	…
A	41
B	42
C	43
…	…

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85

string	codeword
AD	86
DA	87
ABR	88
RAB	89
BRA	8A

Java implementation, LZW compression

```
private static final int R = 256;             // number of input chars
private static final int L = 4096;            // number of codewords = 2^W
private static final int W = 12;              // codeword width

public static void compress()
{
   String input = BinaryStdIn.readString();   // read input as string
   TST<Integer> st = new TST<Integer>();      // trie data structure

   for (int i = 0; i < R; i++)                // codewords for single chars
      st.put("" + (char) i, i);
   int code = R+1;                            // R is codeword for EOF

   while (input.length() > 0)
   {
      String s = st.longestPrefixOf(input);
      BinaryStdOut.write(st.get(s), W);       // write W-bit codeword for s
      int t = s.length();
      if (t < input.length() && code < L)
         st.put(input.substring(0, t + 1), code++);  // add new codeword
      input = input.substring(t);
   }
   BinaryStdOut.write(R, W);                  // write last codeword
   BinaryStdOut.close();
}
```

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output													

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

val_{old} = A
x = 42
s = B
c = B
$next_{code}$ = 81
val_{new} = B

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A												

string	codeword
…	…
A	41
B	**42**
C	43
D	44
…	…
R	**52**
…	…

Initial set of character codewords

string	codeword
AB	81

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

val_{old} = B
x = 52
s = R
c = R
$next_{code}$ = 82
val_{new} = R

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B											

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

val_{old} = R
x = 41
s = A
c = A
$next_{code}$ = 83
val_{new} = A

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R										

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

```
val_old   = A
x         = 43
s         = C
c         = C
next_code = 84
val_new   = C
```

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A									

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

```
val_old   = C
x         = 41
s         = A
c         = A
next_code = 85
val_new   = A
```

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C								

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

```
val_old   = A
x         = 44
s         = D
c         = D
next_code = 86
val_new   = D
```

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A							

string	codeword
…	…
A	41
B	42
C	43
D	**44**
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	**81**
BR	82
RA	83
AC	84
CA	85
AD	86

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

```
val_old   = D
x         = 81
s         = AB
c         = A
next_code = 87
val_new   = AB
```

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A	D						

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

```
val_old    = AB
x          = 83
s          = RA
c          = R
next_code  = 88
val_new    = RA
```

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A	D	AB					

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	**82**
RA	**83**
AC	84
CA	85
AD	86
DA	87
ABR	88

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

val_{old} = RA
x = 82
s = BR
c = B
$next_{code}$ = 89
val_{new} = BR

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A	D	AB	RA				

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	**82**
RA	83
AC	84
CA	85
AD	86
DA	87
ABR	**88**
RAB	89

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

val_{old} = BR
x = 88
s = ABR
c = A
$next_{code}$ = 8A
val_{new} = RA

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A	D	AB	RA	BR			

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87
ABR	88
RAB	89
BRA	8A

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

val_{old} = ABR
x = 41
s = A
c = A
$next_{code}$ = 8B
val_{new} = RA

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A	D	AB	RA	BR	ABR		

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87
ABR	88
RAB	89
BRA	8A
ABRA	8B

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

val_{old} = A
x = 80
s =
c =
$next_{code}$ =
val_{new} =

input	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A	D	AB	RA	BR	ABR	A	

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

Initial set of character codewords

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87
ABR	88
RAB	89
BRA	8A
ABRA	8B

Mappings found during compression

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

val_{old} =
x =
s =
c =
$next_{code}$ =
val_{new} =

Expansion data structure

- For a W-bit codeword, look up string value
  - An array of size 2^W

string	codeword
…	…
A	41
B	42
C	43
D	44
…	…
R	52
…	…

string	codeword
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86
DA	87
ABR	88
RAB	89
BRA	8A
ABRA	8B

- Expansion
  - Write the current string `val`
  - Read codeword `x` from input
  - Set `s` to string for codeword `x`
  - Set next unassigned codeword to `val+c` where `c` is 1st char in `s`
  - Set `val=s`

A tricky case: compression

ABABABA

input	A	B	A	B	A	B	A
matches							
codeword							

string	codeword
…	…
A	41
B	42
C	43
…	…

Initial set of codewords

string	codeword

Mappings found during compression

A tricky case: compression

ABABABA

input	A	B	A	B	A	B	A
matches	A						
codeword	41						

string	codeword
…	…
A	41
B	42
C	43
…	…

Initial set of codewords

string	codeword
AB	81

Mappings found during compression

A tricky case: compression

ABABABA

input	A	B	A	B	A	B	A
matches	A	B					
codeword	41	42					

string	codeword
…	…
A	41
B	42
C	43
…	…

Initial set of codewords

string	codeword
AB	81
BA	82

Mappings found during compression

A tricky case: compression

ABABABA

input	A	B	A	B	A	B	A
matches	A	B	AB				
codeword	41	42	81				

string	codeword
…	…
A	41
B	42
C	43
…	…

Initial set of codewords

string	codeword
AB	81
BA	82
ABA	83

Mappings found during compression

A tricky case: compression

ABABABA

input	A	B	A	B	A	B	A
matches	A	B	AB		ABA		
codeword	41	42	81		83		

Notice right after adding entry for "AB" the subsequent text had "AB" as a prefix.

string	codeword
…	…
A	41
B	42
C	43
…	…

Initial set of codewords

string	codeword
AB	81
BA	82
ABA	83

Mappings found during compression

A tricky case: expansion

ABABABA

input	41	42	81	83	80
output					

string	codeword
…	…
A	41
B	42
C	43
…	…

Initial set of codewords

string	codeword

Mappings found during compression

A tricky case: expansion

ABABABA

input	41	42	81	83	80
output	A				

string	codeword
…	…
A	41
B	42
C	43
…	…

Initial set of codewords

string	codeword
AB	81

Mappings found during compression

A tricky case: expansion

ABABABA

input	41	42	81	83	80
output	A	B			

string	codeword
…	…
A	41
B	42
C	43
…	…

Initial set of codewords

string	codeword
AB	81
BA	82

Mappings found during compression

A tricky case: expansion

ABABABA

input	41	42	81	83	80
output	A	B	AB		

We need to know the first character of 83 in order to enter 83 into the table!

Fix: First character of 83 must be same as first letter of current codeword 81, i.e. "A".

string	codeword
…	…
A	41
B	42
C	43
…	…

Initial set of codewords

string	codeword
AB	81
BA	82

Mappings found during compression

A tricky case: expansion

ABABABA

input	41	42	81	83	80
output	A	B	AB	ABA	

string	codeword
…	…
A	41
B	42
C	43
…	…

Initial set of codewords

string	codeword
AB	81
BA	82
ABA	83

Mappings found during compression

Java implementation, LZW expansion

```
private static final int R = 256;            // number of input chars
private static final int L = 4096;           // number of codewords = 2^W
private static final int W = 12;             // codeword width

public static void expand()
{
   String[] st = new String[L];
   int i; // next available codeword value

   // initialize symbol table with all 1-character strings
   for (i = 0; i < R; i++)
      st[i] = "" + (char) i;
   st[i++] = "";                            // (unused) lookahead for EOF

   int codeword = BinaryStdIn.readInt(W);
   String val = st[codeword];

   while (true)
   {
      BinaryStdOut.write(val);
      codeword = BinaryStdIn.readInt(W);
      if (codeword == R) break;
         String s = st[codeword];
      if (i == codeword) s = val + val.charAt(0);   // special case hack
      if (i < L) st[i++] = val + s.charAt(0);
         val = s;
   }
   BinaryStdOut.close();
}
```

LZW decisions

- How big of a symbol table?
  - How long is message?
  - Whole message similar model?
  - Many variations...
- What to do when symbol table fills up?
  - Throw away and start over (e.g. GIF)
  - Throw away when not effective (e.g. Unix compress)
  - Many variations...
- Put longer substrings in symbol table?
  - Many variations...

LZW variants

- Lempel-Ziv variants
  - LZ77, published Lempel & Ziv, 1977, not patented
    - PNG
  - LZ78, published Lempel & Ziv in 1978, patented
  - LZW, Welch extension to LZ78, patented (expired 2003)
    - GIF, TIFF, Pkzip
  - Deflate = LZ77 variant + Huffman coding
    - 7zip, gzip, jar, PDF

Burn All GIFs

Some experiments

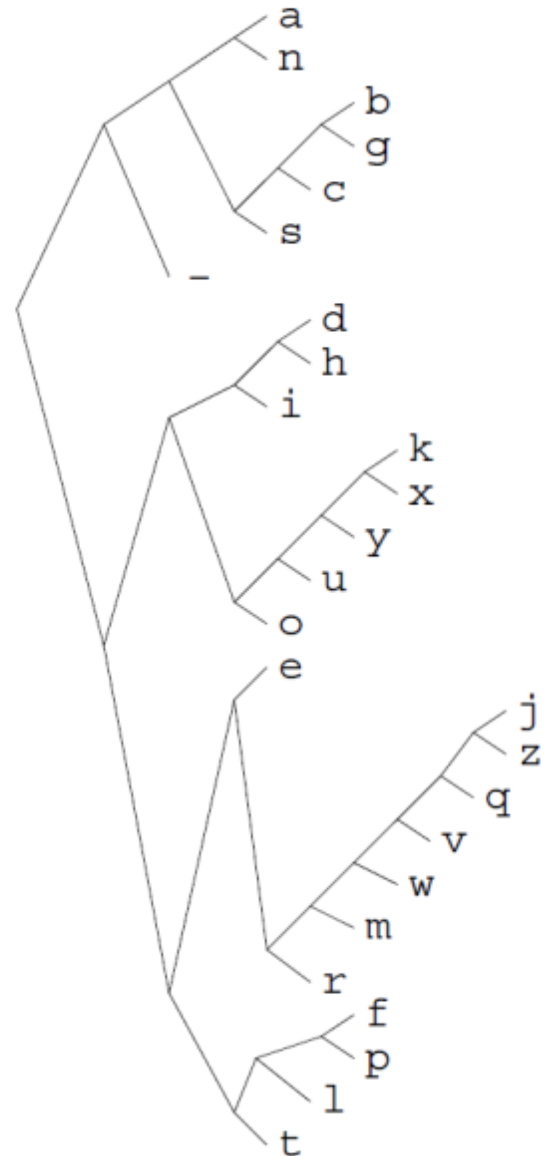
- Compressing Moby Dick
  - Using Java programs for LZW and Huffman
    - Different dictionary sizes for LZW or resetting when full
  - Compared to zip and gzip

```
1,191,463   mobydick.txt
  485,561   mobydick.gz
  485,790   mobydick.zip
  398,480   mobydick.xz
  371,394   mobydick.bz2

  667,651   mobydick.huff
  597,060   mobydick.lzw
  814,578   mobydick.huff.lzw
  592,830   mobydick.lzw.huff
  682,400   mobydick.lzw.reset

  541,261   mobydick.lzw14
  521,700   mobydick.lzw15
  503,050   mobydick.lzw16
  501,193   mobydick.lzw16.huff
  521,700   mobydick.lzw17
  514,393   mobydick.lzw18
  571,548   mobydick.lzw20
```

Enter statistical coding…

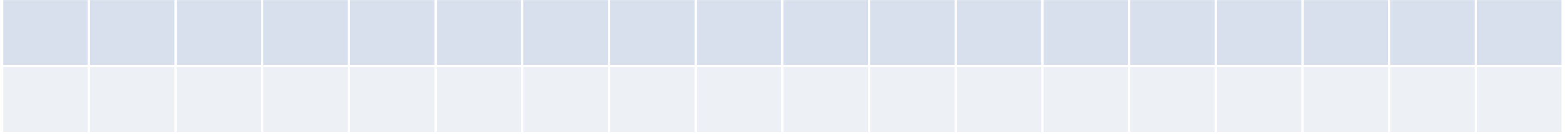
- Natural language quite predictable
  - ~ 1 bit of entropy per symbol
  - Huffman coding still requires 1-bit min per symbol
    - We're forced to use an integral number of bits
  - Dictionary-based (LZW and friends)
    - Memorizes sequences, but no notion of frequency
- Statistical coding
  - Use long, specific context for prediction
    - P(A | The_United_State_of_) = ?
  - Blend knowledge using contexts of different lengths
  - Adaptive: model updates and change as text seen
    - Often after every letter!

a_i	p_i	$\log_2 \frac{1}{p_i}$	l_i	$c(a_i)$
a	0.0575	4.1	4	0000
b	0.0128	6.3	6	001000
c	0.0263	5.2	5	00101
d	0.0285	5.1	5	10000
e	0.0913	3.5	4	1100
f	0.0173	5.9	6	111000
g	0.0133	6.2	6	001001
h	0.0313	5.0	5	10001
i	0.0599	4.1	4	1001
j	0.0006	10.7	10	1101000000
k	0.0084	6.9	7	1010000
l	0.0335	4.9	5	11101
m	0.0235	5.4	6	110101
n	0.0596	4.1	4	0001
o	0.0689	3.9	4	1011
p	0.0192	5.7	6	111001
q	0.0008	10.3	9	110100001
r	0.0508	4.3	5	11011
s	0.0567	4.1	4	0011
t	0.0706	3.8	4	1111
u	0.0334	4.9	5	10101
v	0.0069	7.2	8	11010001
w	0.0119	6.4	7	1101001
x	0.0073	7.1	7	1010001
y	0.0164	5.9	6	101001
z	0.0007	10.4	10	1101000001
–	0.1928	2.4	2	01



MacKay, D.J.C. Information Theory, Inference, and Learning Algorithms.

Guess the phrase



A simple unigram model of English

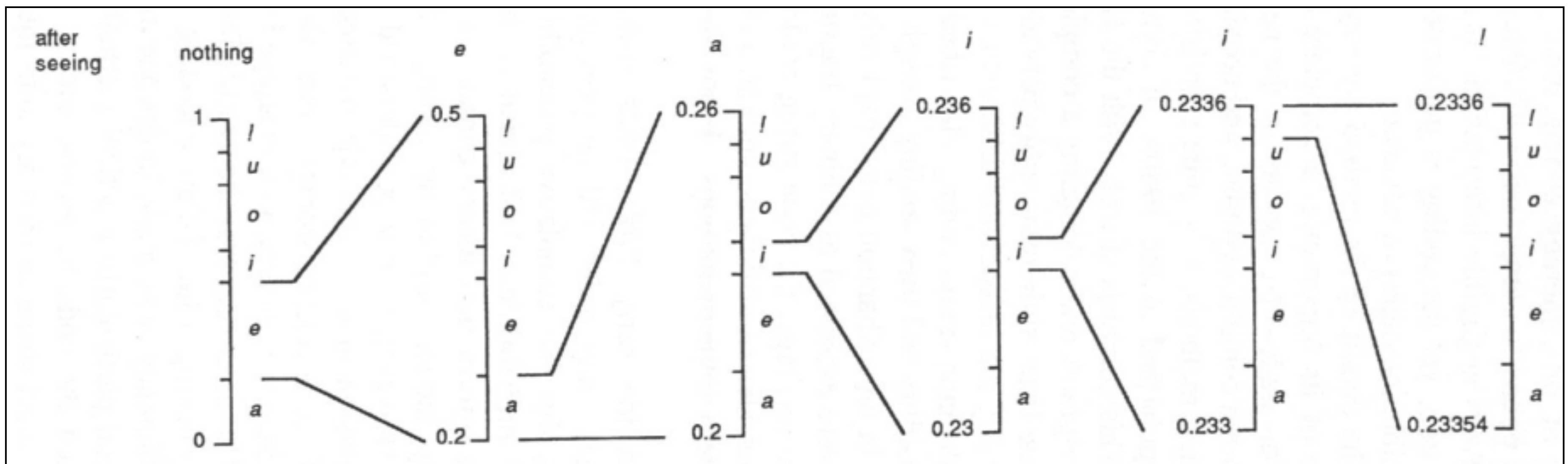
'	0.0071063
-	0.0000001
.	0.0000384
</s>	0.0368291
<sp>	0.1653810
a	0.0595248
b	0.0112864
c	0.0174441
d	0.0282733
e	0.0890307
f	0.0127512
g	0.0213974
h	0.0403836
i	0.0586443
j	0.0018080
k	0.0117826

l	0.0343399
m	0.0247835
n	0.0490316
o	0.0762119
p	0.0134453
q	0.0003078
r	0.0408972
s	0.0433802
t	0.0680194
u	0.0273347
v	0.0083669
w	0.0210079
x	0.0010829
y	0.0295698
z	0.0005395

From text to a real number

- Arithmetic coding
  - Message represented by real interval in [0, 1)
  - More precise interval = specifies more bits
    - e.g. [0.58272722, 0.58272724) = "it was the best of times"
    - Or any number in that interval, 0.58272723
- Example
  - Alphabet = `{aeiou!}`
  - Transmission = `eaii!`

Symbol	Probability	Range
a	0.2	[0.0, 0.2)
e	0.3	[0.2, 0.5)
i	0.1	[0.5, 0.6)
o	0.2	[0.6, 0.8)
u	0.1	[0.8, 0.9)
!	0.1	[0.9, 1.0)



Bell, Cleary, Witten. Text Compression.

Transmission = eaii!

After seeing	Range
	[0.0, 1.0]
e	[0.2, 0.5)
a	[0.2, 0.26)
i	[0.23, 0.236)
i	[0.233, 0.2336)
!	[0.23354, 0.2336)

Symbol	Probability	Range
a	0.2	[0.0, 0.2)
e	0.3	[0.2, 0.5)
i	0.1	[0.5, 0.6)
o	0.2	[0.6, 0.8)
u	0.1	[0.8, 0.9)
!	0.1	[0.9, 1.0)

Note: Encoder/decoder agree on symbol to terminate message, here we use !

Context modeling

- Prediction by Partial Match (PPM)
  - P(next symbol) depends on previous symbol(s)
  - Blending for dealing with 0-frequency problem
  - Easy to build as adaptive
    - Learn from the text as you go
    - No trie to send as in Huffman coding
  - 1984, but still competitive for text compression
    - Various variants, PPM-A/B/C/D/Z/*
    - Implemented in 7-zip, open source packages, PPM for XML, PPM for executables, …

PPM

- An alphabet A with q symbols
- Order = How many previous symbols to use

  …

  2 = condition on 2 previous symbols, $P(x_n \mid x_{n-1}, x_{n-2})$

  1 = condition on 1 previous symbol, $P(x_n \mid x_{n-1})$

  0 = condition on current symbol, $P(x_n)$

  -1 = uniform over the alphabet, $P(x_n) = 1/q$

- For a given order:
  - Probability of next symbol based on counting occurrences given prior context

$$p_o(\phi) = \frac{c_o(\phi)}{C_o}.$$

Escape probabilities

- Need weights to blend probabilities
  - Compute based on "escape" probability
  - Allocate some mass in each order for when a lower-order model should make prediction instead
  - Method A:
    - Add one to C_0: count of characters seen in this context
    - $e_o = 1 / (C_0 + 1)$
  - Method D:
    - u = number of unique characters seen in this context
    - $e_o = (u / 2) / C_0$

TABLE 6-1 CALCULATION OF BLENDED PROBABILITIES (SIMPLE SCHEME AND ESCAPE METHOD A) FOR THE MESSAGE "cacbcaabca"

	Context	Counts	Predictions — $p_o(\phi)$ $c_o(\phi)$										Weight	Escape
o		C_o	a		b		c		d		e		w_o	e_o
4	abca:	0	—	0	—	0	—	0	—	0	—	0	0	—
3	bca:	1	1	1	0	0	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$
2	ca:	2	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	1	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$
1	a:	3	$\frac{1}{3}$	1	$\frac{1}{3}$	1	$\frac{1}{3}$	1	0	0	0	0	$\frac{1}{8}$	$\frac{1}{4}$
0	:	10	$\frac{4}{10}$	4	$\frac{2}{10}$	2	$\frac{4}{10}$	4	0	0	0	0	$\frac{5}{132}$	$\frac{1}{11}$
−1	—	—	$\frac{1}{5}$	—	$\frac{1}{5}$	—	$\frac{1}{5}$	—	$\frac{1}{5}$	—	$\frac{1}{5}$	—	$\frac{1}{264}$	0
Blended probabilities			$\frac{956}{1320}$		$\frac{66}{1320}$		$\frac{296}{1320}$		$\frac{1}{1320}$		$\frac{1}{1320}$			

Bell, Cleary, Witten. Text Compression.

$$p(\phi) = \sum_{o=-1}^{m} w_o p_o(\phi).$$

Lossless compression benchmarks

year	scheme	bits /char
1967	ASCII	7.00
1950	Huffman	4.70
1977	LZ77	3.94
1984	LZMW	3.32
1987	LZH	3.30
1987	move-to-front	3.24
1987	LZB	3.18
1987	gzip	2.71
1988	PPMC	2.48
1994	PPM	2.34
1995	Burrows-Wheeler	2.29
1997	BOA	1.99
1999	RK	1.89

Data compression using Calgary corpus

Compressing Moby Dick

```
1,191,463   mobydick.txt
  485,561   mobydick.gz
  485,790   mobydick.zip
  398,480   mobydick.xz
  371,394   mobydick.bz2

  667,651   mobydick.huff
  597,060   mobydick.lzw
  814,578   mobydick.huff.lzw
  592,830   mobydick.lzw.huff
  682,400   mobydick.lzw.reset

  541,261   mobydick.lzw14
  521,700   mobydick.lzw15
  503,050   mobydick.lzw16
  501,193   mobydick.lzw16.huff
  521,700   mobydick.lzw17
  514,393   mobydick.lzw18
  571,548   mobydick.lzw20

  325,378   mobydick.ppmd

  306,708   mobydick.zpaq
```

Summary

- Dictionary-based compression
  - LZW and variants
  - Memorizes sequences in data
- Statistical coding
  - Language model produces probabilities
  - Probability sequence defines point on [0, 1)
  - Use arithmetic coding to convert to bits
  - Prediction by Partial Match (PPM)