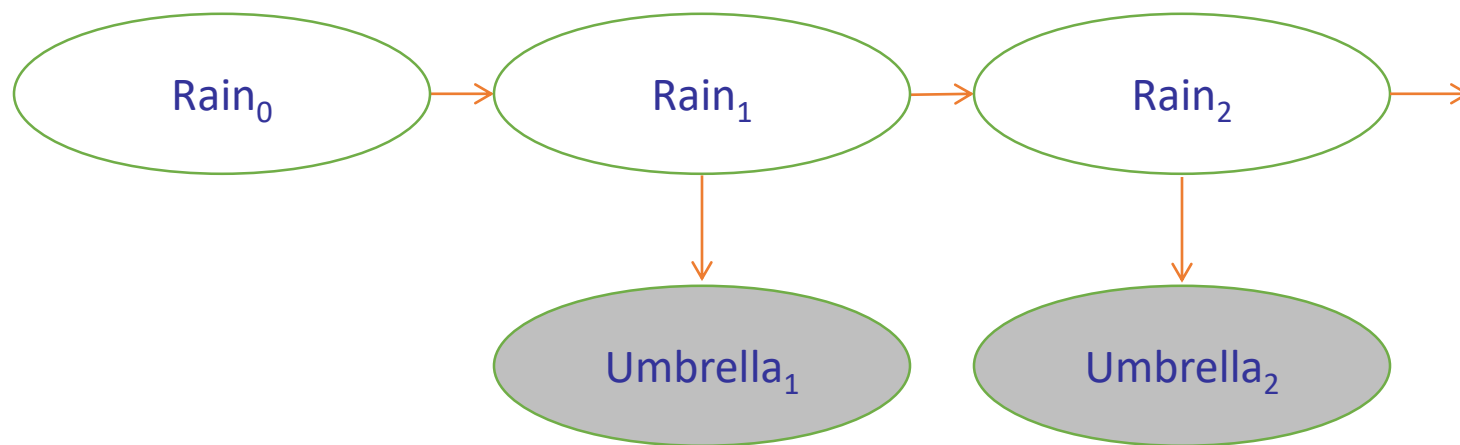


Sad Grad Student HMM: Example

Example: Weather HMM



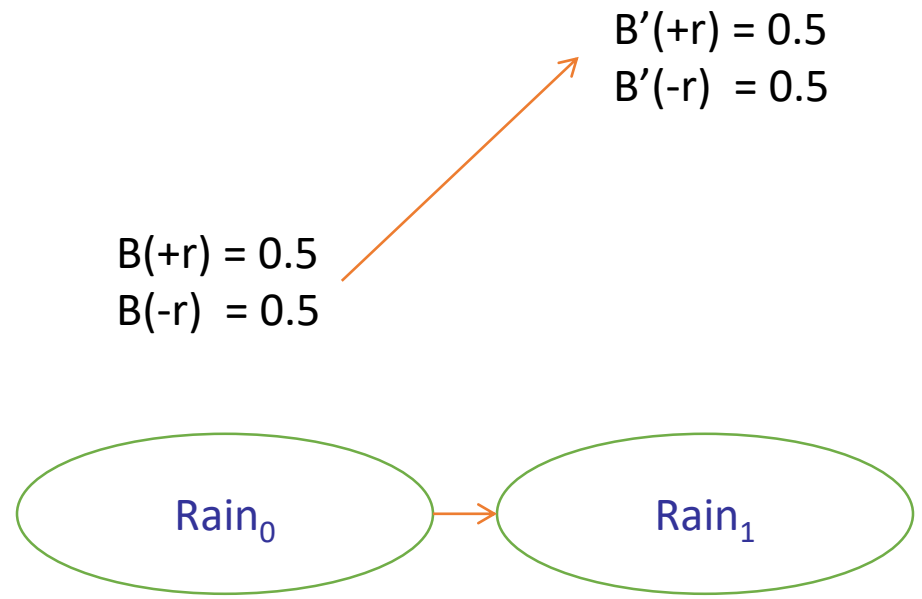
$$\begin{array}{l}
 B(+r) = 0.5 \\
 B(-r) = 0.5
 \end{array}
 \begin{array}{l}
 \nearrow \\
 \downarrow \\
 \nearrow \\
 \downarrow
 \end{array}
 \begin{array}{l}
 B'(+r) = 0.5 \\
 B'(-r) = 0.5 \\
 B(+r) = 0.818 \\
 B(-r) = 0.182
 \end{array}
 \begin{array}{l}
 \nearrow \\
 \downarrow \\
 \nearrow \\
 \downarrow
 \end{array}
 \begin{array}{l}
 B'(+r) = 0.627 \\
 B'(-r) = 0.373 \\
 B(+r) = 0.883 \\
 B(-r) = 0.117
 \end{array}$$



R_t	R_{t+1}	$P(R_{t+1} R_t)$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

R_t	U_t	$P(U_t R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

Example: Weather HMM



Time Passes:

$$B'(X_{t+1}) = \sum_{x_t} P(X_{t+1} | x_t) B(x_t)$$

Observation:

$$B(X_{t+1}) \propto_{x_{t+1}} P(e_{t+1} | X_{t+1}) B'(x_{t+1})$$

Time Passes:

$$\begin{aligned}
 P(\text{rain} | \text{sun}) B(\text{sun}) + P(\text{rain} | \text{rain}) B(\text{rain}) &= 0.3(0.5) + 0.7(0.5) \\
 &= 0.15 + 0.35 = 0.5
 \end{aligned}$$

$$\begin{aligned}
 P(\text{sun} | \text{sun}) B(\text{sun}) + P(\text{sun} | \text{rain}) B(\text{rain}) &= 0.7(0.5) + 0.3(0.5) \\
 &= 0.35 + 0.15 = 0.5
 \end{aligned}$$

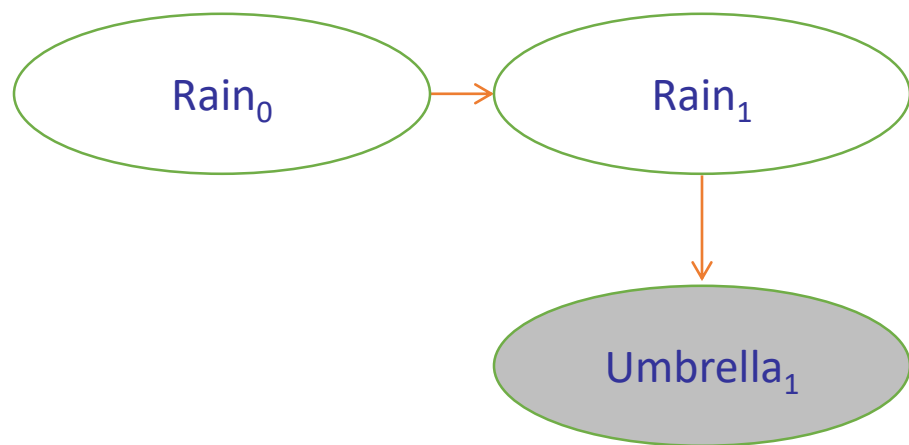
R_t	R_{t+1}	$P(R_{t+1} R_t)$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

R_t	U_t	$P(U_t R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

Example: Weather HMM



$$\begin{array}{l}
 B(+r) = 0.5 \\
 B(-r) = 0.5
 \end{array}
 \begin{array}{l}
 \nearrow \\
 \searrow
 \end{array}
 \begin{array}{l}
 B'(+r) = 0.5 \\
 B'(-r) = 0.5
 \end{array}
 \begin{array}{l}
 \searrow \\
 \nearrow
 \end{array}
 \begin{array}{l}
 B(+r) = 0.818 \\
 B(-r) = 0.182
 \end{array}$$



Time Passes:

$$B'(X_{t+1}) = \sum_{x_t} P(X_{t+1} | x_t) B(x_t)$$

Observation:

$$B(X_{t+1}) \propto_{x_{t+1}} P(e_{t+1} | X_{t+1}) B'(x_{t+1})$$

Observe:

$$P(+um | rain) B'(rain) = 0.9(0.5) = 0.45$$

$$P(+um | sun) B'(sun) = 0.2(0.5) = 0.1$$

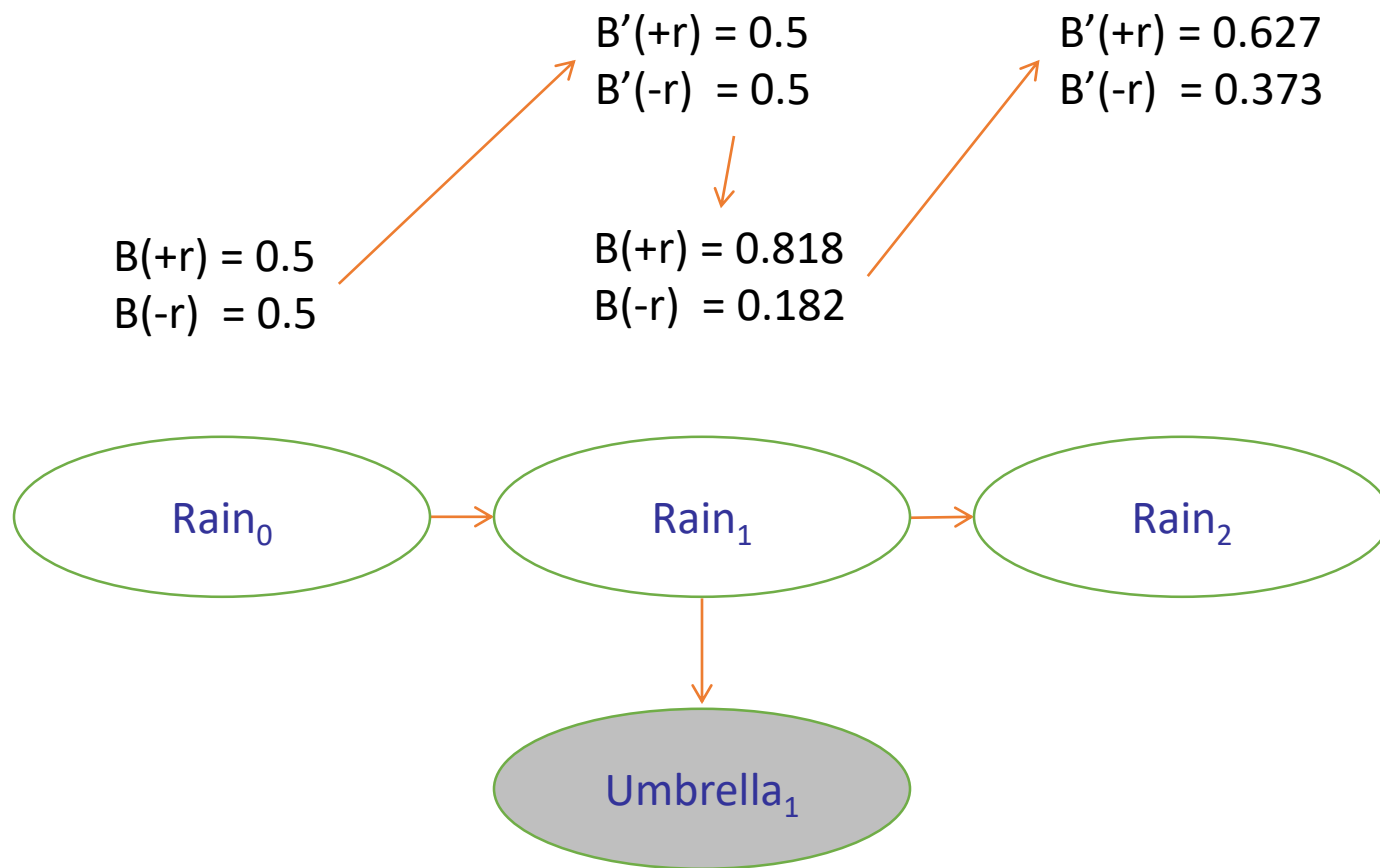
$$Z = 0.1 + 0.45 = 0.55$$

$$B(rain) = 0.45/0.55 = 0.818, B(sun) = 0.1/0.55 = 0.182$$

R_t	R_{t+1}	$P(R_{t+1} R_t)$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

R_t	U_t	$P(U_t R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

Example: Weather HMM



Time Passes:
 $B'(X_{t+1}) = \sum_{x_t} P(X_{t+1} | x_t) B(x_t)$

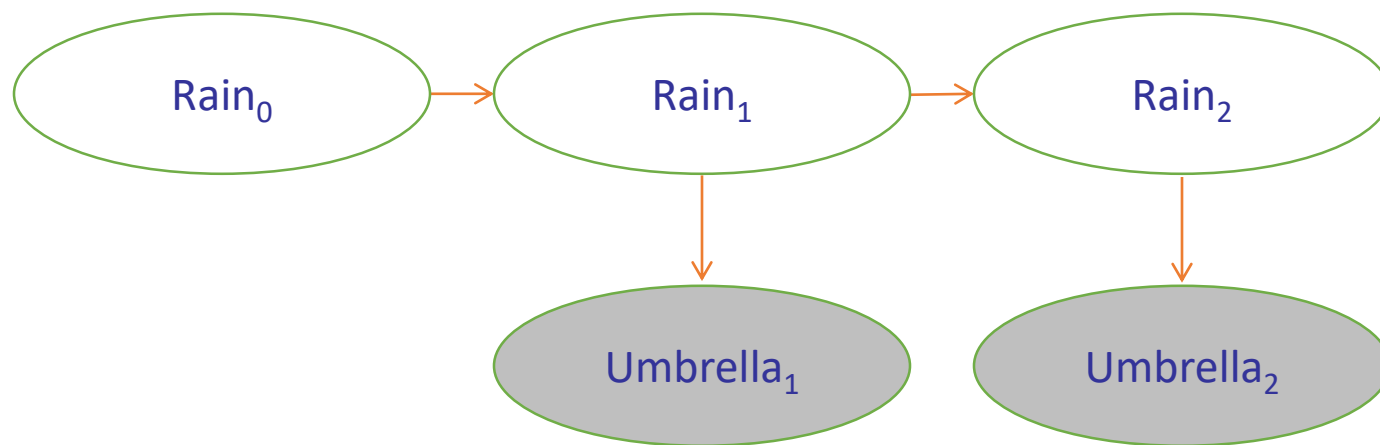
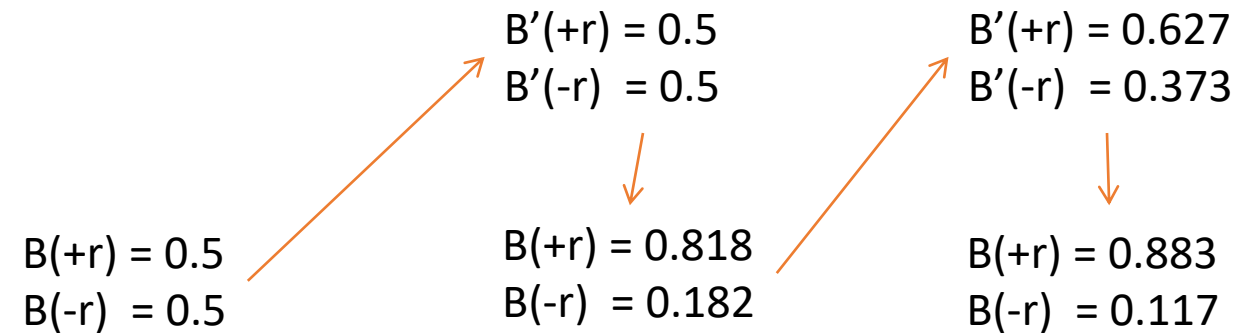
Time Passes:
 $P(\text{rain} | \text{sun}) B(\text{sun})$
 $+ P(\text{rain} | \text{rain}) B(\text{rain}) = 0.3(0.182) + 0.7(0.818)$
 $= 0.0546 + 0.5726 = 0.627$

$P(\text{sun} | \text{sun}) B(\text{sun})$
 $+ P(\text{sun} | \text{rain}) B(\text{rain}) = 0.7(0.182) + 0.3(0.818)$
 $= 0.1274 + 0.2454 = 0.373$

R_t	R_{t+1}	$P(R_{t+1} R_t)$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

R_t	U_t	$P(U_t R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

Example: Weather HMM



Observation:

$$B(X_{t+1}) \propto_{x_{t+1}} P(e_{t+1} | X_{t+1}) B'(x_{t+1})$$

Observe:

$$P(+um | rain) B'(rain) = 0.9(0.627) = 0.5643$$

$$P(+um | sun) B'(sun) = 0.2(0.373) = 0.0746$$

$$Z = 0.5643 + 0.0746 = 0.6389$$

$$B(rain) = 0.5643 / 0.6389 = 0.883$$

$$B(sun) = 0.0746 / 0.6389 = 0.117$$

R_t	R_{t+1}	$P(R_{t+1} R_t)$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

R_t	U_t	$P(U_t R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8