ASSIGNMENT 4

The goal of this assignment is to practice using formal logic. You will translate a series of sentences into first order logic and then use three different techniques to see if you can prove an assertion.

Part 1: Translating English into Formal Logic

For the first part, you will translate the sentences below into first order logic. These came from the famous mathematician (and more famous writer) Lewis Carroll. You might want to check out the puzzles at the [Lewis Carrol Puzzles website] to see how some of them are approached.

No kitten, that loves fish, is unteachable.
¬∃x, (kitten(x) ∧ lovesFish(x)) ∧ ¬teachable(x)
∀x, ¬((kitten(x) ∧ lovesFish(x)) ∧ ¬teachable(x))
∀x, ¬(kitten(x) ∧ lovesFish(x)) ∨ teachable(x)
∀x, kitten(x) ∧ lovesFish(x) ⇒ teachable(x) Horn form
∀x, ¬kitten(x) ∨ ¬lovesFish(x) ∨ teachable(x) CNF

No kitten without a tail will play with a gorilla.
¬∃x, kitten(x) ∧ ¬tail(x) ∧ playwGorilla(x)
∀x, ¬(kitten(x) ∧ ¬tail(x) ∧ playwGorilla(x))
∀x, ¬(kitten(x) ∧ ¬tail(x)) ∨ ¬playwGorilla(x)
∀x, kitten(x) ∧ ¬tail(x) ⇒ ¬playwGorilla(x) Horn form
∀x, ¬kitten(x) ∨ tail(x) ∨ ¬playwGorilla(x) CNF

Kittens with whiskers always love fish.
∀x, kitten(x) ∧ hasWhiskers(x) ⇒ lovesFish(x) Horn form
∀x, ¬(kitten(x) ∧ hasWhiskers(x)) ∨ lovesFish(x)
∀x, ¬kitten(x) ∨ ¬hasWhiskers(x) ∨ lovesFish(x) CNF

No teachable kitten has green eyes.
¬∃x, kitten(x) ∧ teachable(x) ∧ greenEyes(x)
∀x, ¬(kitten(x) ∧ teachable(x) ∧ greenEyes(x))
∀x, ¬(kitten(x) ∧ teachable(x)) ∨ ¬greenEyes(x)
∀x, kitten(x) ∧ teachable(x) ⇒ ¬greenEyes(x) Horn form
∀x, ¬kitten(x) ∨ ¬teachable(x) ∨ ¬greenEyes(x) CNF

No kittens have tails unless they have whiskers.
¬∃x, kitten(x) ∧ tail(x) ∧ ¬hasWhiskers(x)
∀x, ¬(kitten(x) ∧ tail(x) ∧ ¬hasWhiskers(x))
∀x, ¬(kitten(x) ∧ tail(x)) ∨ hasWhiskers(x)
∀x, kitten(x) ∧ tail(x) ⇒ hasWhiskers(x) Horn form
∀x, ¬kitten(x) ∨ ¬tail(x) ∨ hasWhiskers(x) CNF

Part 2: Inference in First Order Logic

Let's say we also know:

My kitten who has green eyes does not love fish.
∃y, kitten(y) ∧ greenEyes(y) ∧ ¬lovesFish(y)
kitten(Kiki) ∧ greenEyes(Kiki) ∧ ¬lovesFish(Kiki)

Using three approaches, try to prove the following:

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My kitten is unteachable.
∃y, kitten(y) ∧ ¬teachable(y)
kitten(Kiki) ∧ ¬teachable(Kiki)
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A. Forward Chaining. Use forward chaining to try to prove the previous assertion. Remember, that in order to use forward (or backward) chaining, the knowledge base must be in Horn form. So start by translating your KB into Horn clauses. Show your work in doing the translation, and then as you do the inference, show the logical progression so I can see how you did your reasoning.

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∀x, kitten(x) ∧ lovesFish(x) ⇒ teachable(x)          Horn form
∀x, kitten(x) ∧ ¬tail(x) ⇒ ¬playwGorilla(x)          Horn form
∀x, kitten(x) ∧ hasWhiskers(x) ⇒ lovesFish(x)        Horn form
∀x, kitten(x) ∧ teachable(x) ⇒ ¬greenEyes(x)         Horn form
∀x, kitten(x) ∧ tail(x) ⇒ hasWhiskers(x)             Horn form
kitten(Kiki)
greenEyes(Kiki)
¬lovesFish(Kiki)

Prove ¬teachable(Kiki)

Subst{x/Kiki}, all kitten(x) clauses are satisfied

1.  lovesFish(Kiki) ⇒ teachable(Kiki)          Horn form
2.  ¬teachable(Kiki) ⇒ ¬lovesFish(Kiki)        Contraposition
3.  ¬tail(Kiki) ⇒ ¬playwGorilla(Kiki)          Horn form
4.  playwGorilla(Kiki) ⇒ tail(Kiki)            Contraposition
5.  hasWhiskers(Kiki) ⇒ lovesFish(Kiki)        Horn form
6.  ¬lovesFish(Kiki) ⇒ ¬hasWhiskers(Kiki)      Contraposition
7.  teachable(Kiki) ⇒ ¬greenEyes(Kiki)         Horn form
8.  greenEyes(Kiki) ⇒ ¬teachable(Kiki)         Contraposition
9.  tail(Kiki) ⇒ hasWhiskers(Kiki)             Horn form
10. ¬hasWhiskers(Kiki) ⇒ ¬tail(Kiki)           Contraposition
11. greenEyes(Kiki)
12. ¬lovesFish(Kiki)

Modus ponens, 12 and 6, add to KB:
13. ¬hasWhiskers(Kiki)
Modus ponens, 13 and 10, add to KB:
14. ¬tail(Kiki)
Modus ponens, 14 and 3, add to KB:
15. ¬playwGorilla(Kiki)
Modus ponens, 11 and 8, add to KB:
¬teachable(Kiki)                                PROVEN
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B. Backward Chaining. Now use backward chaining to try to prove the previous assertion. You can use your KB from the previous part, but I do want to see all your steps in doing the backward chaining.

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∀x, kitten(x) ∧ lovesFish(x) ⇒ teachable(x)          Horn form
∀x, kitten(x) ∧ ¬tail(x) ⇒ ¬playwGorilla(x)          Horn form
∀x, kitten(x) ∧ hasWhiskers(x) ⇒ lovesFish(x)        Horn form
∀x, kitten(x) ∧ teachable(x) ⇒ ¬greenEyes(x)         Horn form
∀x, kitten(x) ∧ tail(x) ⇒ hasWhiskers(x)             Horn form
kitten(Kiki)
greenEyes(Kiki)
¬lovesFish(Kiki)

Prove ¬teachable(Kiki)

Subst{x/Kiki}, all kitten(x) clauses are satisfied

1.  lovesFish(Kiki) ⇒ teachable(Kiki)          Horn form
2.  ¬teachable(Kiki) ⇒ ¬lovesFish(Kiki)        Contraposition
3.  ¬tail(Kiki) ⇒ ¬playwGorilla(Kiki)          Horn form
4.  playwGorilla(Kiki) ⇒ tail(Kiki)            Contraposition
5.  hasWhiskers(Kiki) ⇒ lovesFish(Kiki)        Horn form
6.  ¬lovesFish(Kiki) ⇒ ¬hasWhiskers(Kiki)      Contraposition
7.  teachable(Kiki) ⇒ ¬greenEyes(Kiki)         Horn form
8.  greenEyes(Kiki) ⇒ ¬teachable(Kiki)         Contraposition
9.  tail(Kiki) ⇒ hasWhiskers(Kiki)             Horn form
10. ¬hasWhiskers(Kiki) ⇒ ¬tail(Kiki)           Contraposition
11. greenEyes(Kiki)
12. ¬lovesFish(Kiki)

Clause 8 implies ¬teachable(Kiki), need to prove greenEyes(Kiki).
Clause 11 states greenEyes(Kiki) as given.    PROVEN
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C. Resolution. Finally, use resolution to try to prove the assertion. Remember, that in order to use resolution, your KB must be in conjunctive normal form. So start by translating your KB into CNF. Show me all your work in this step. Then use resolution and show me all of your steps there, also.

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∀x, ¬kitten(x) ∨ ¬lovesFish(x) ∨ teachable(x)        CNF
∀x, ¬kitten(x) ∨ tail(x) ∨ ¬playwGorilla(x)          CNF
∀x, ¬kitten(x) ∨ ¬hasWhiskers(x) ∨ lovesFish(x)     CNF
∀x, ¬kitten(x) ∨ ¬teachable(x) ∨ ¬greenEyes(x)         CNF
∀x, ¬kitten(x) ∨ ¬tail(x) ∨ hasWhiskers(x)         CNF
kitten(Kiki) ∧ greenEyes(Kiki) ∧ ¬lovesFish(Kiki)

Prove ¬teachable(Kiki)

Subst{x/Kiki}
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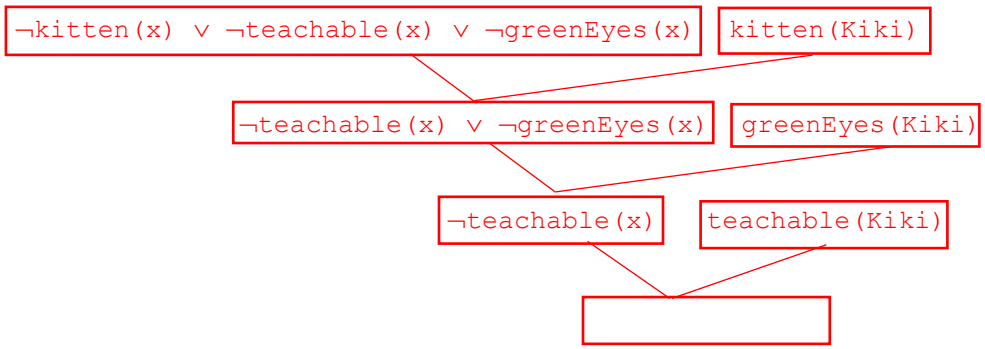

Submission. Submit your work via Moodle.