

# CSCI 446: Artificial Intelligence

## Reinforcement Learning



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[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at <http://ai.berkeley.edu>.]

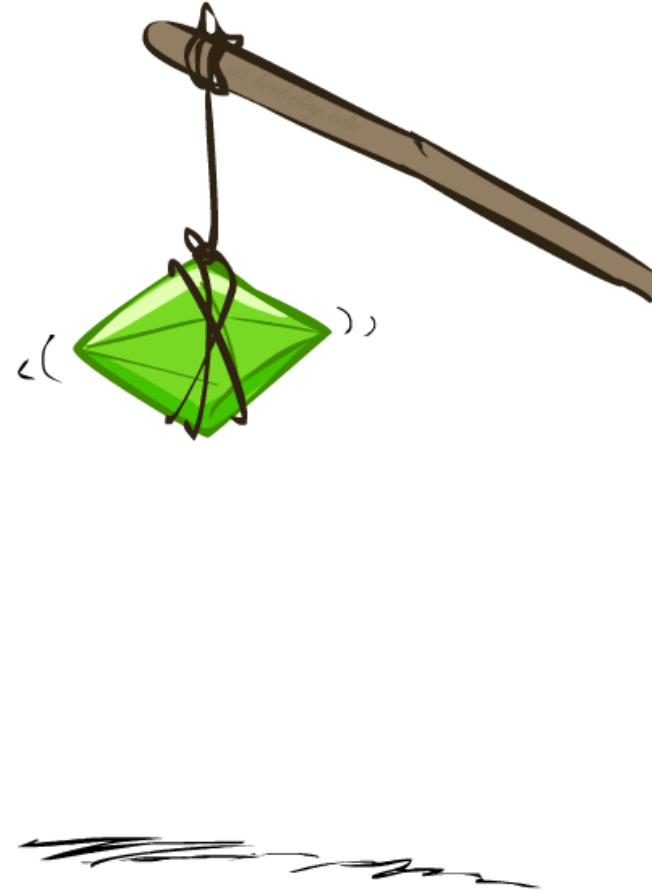
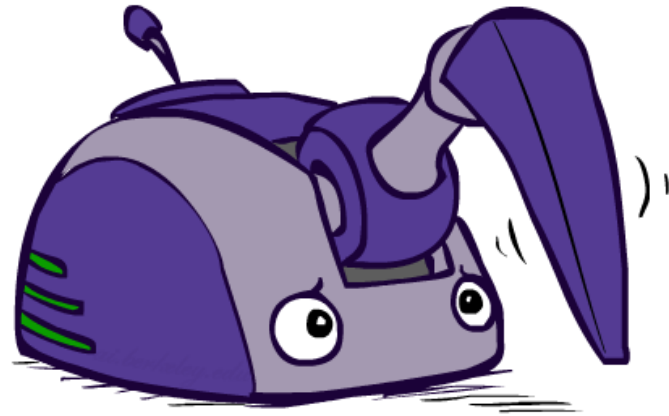
# Today

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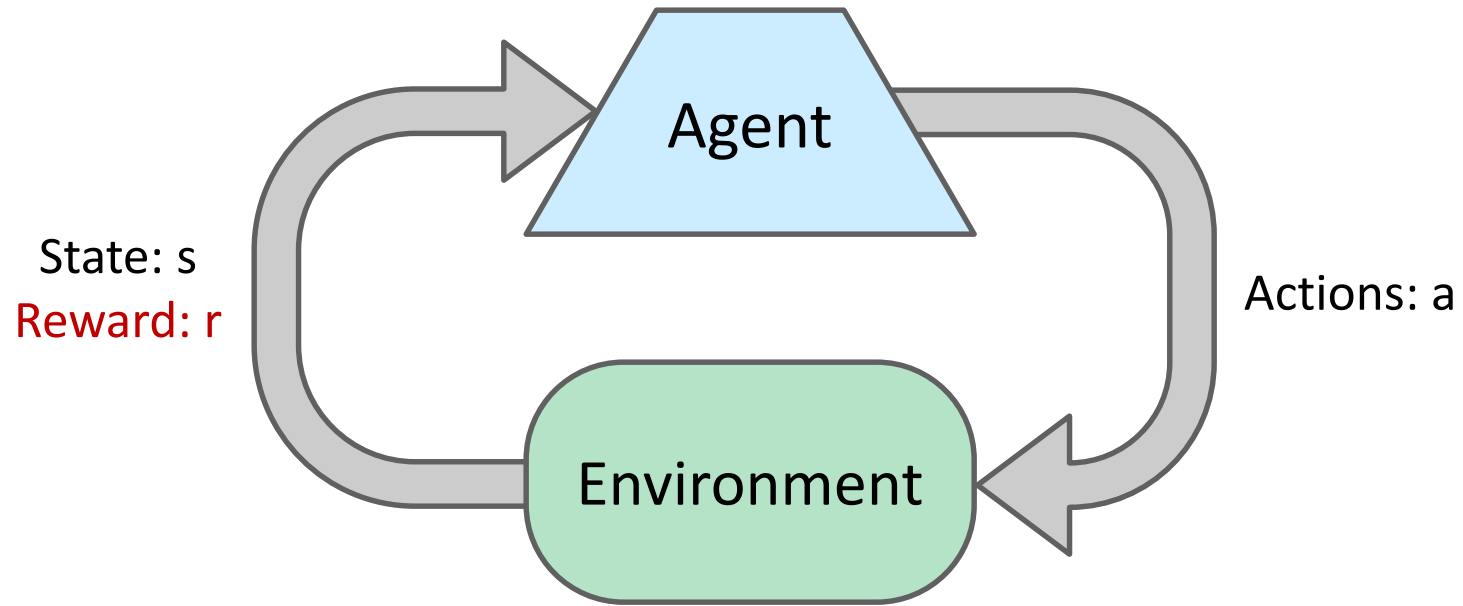
- Reinforcement Learning
- Offline (MDPs) vs Online (RL)
  - Model-Based Learning
  - Model-Free Learning
  - Passive Reinforcement Learning
  - Temporal Difference Learning
  - Active Reinforcement Learning

# Reinforcement Learning

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# Reinforcement Learning



- **Basic idea:**
  - Receive feedback in the form of **rewards**
  - Agent's utility is defined by the reward function
  - Must (learn to) act so as to **maximize expected rewards**
  - All learning is based on observed samples of outcomes!

# Example: Learning to Walk



Initial



A Learning Trial



After Learning [1K Trials]

# Example: Learning to Walk



Initial

# Example: Learning to Walk



Training

# Example: Learning to Walk



Finished

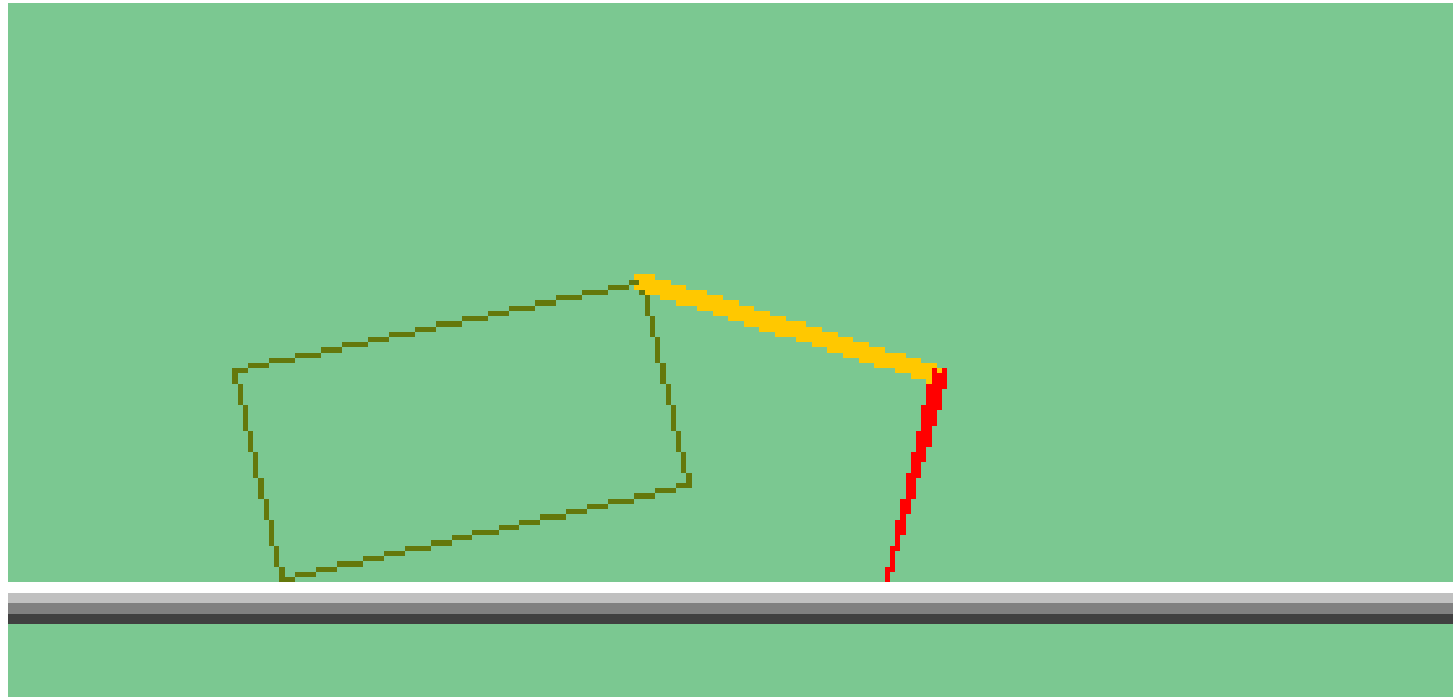


# Example: Toddler Robot



# The Crawler!

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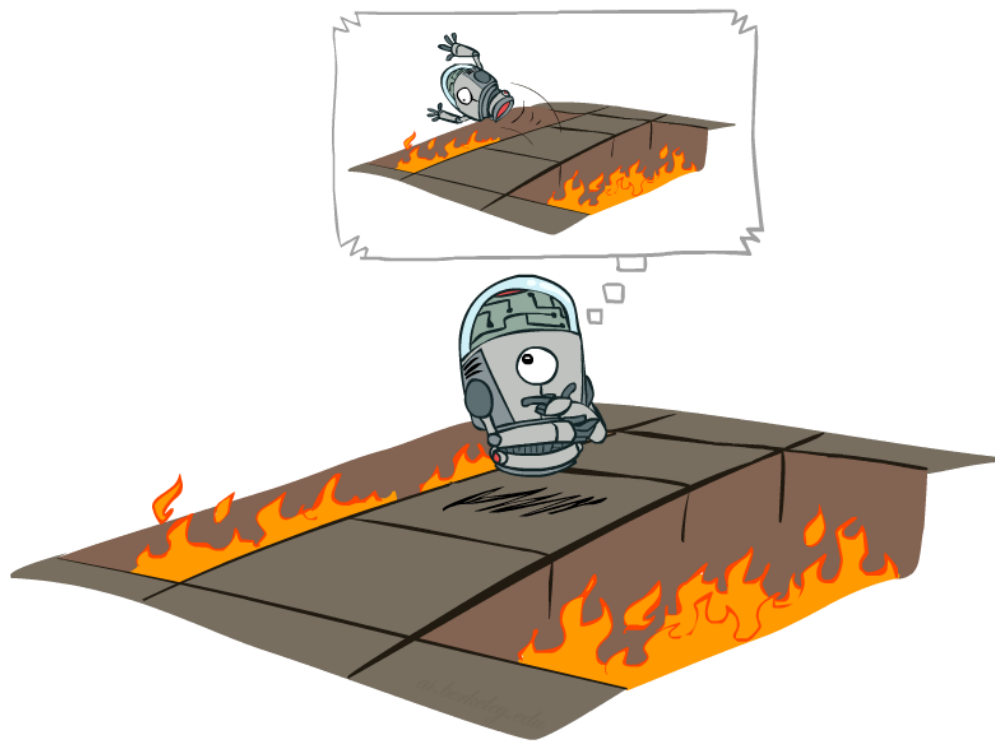


# Reinforcement Learning

- Still assume a Markov decision process (MDP):
  - A set of states  $s \in S$
  - A set of actions (per state)  $A$
  - A model  $T(s,a,s')$
  - A reward function  $R(s,a,s')$
- Still looking for a policy  $\pi(s)$
- New twist: don't know  $T$  or  $R$ 
  - I.e. we don't know which states are good or what the actions do
  - Must actually try actions and states out to learn



# Offline (MDPs) vs. Online (RL)

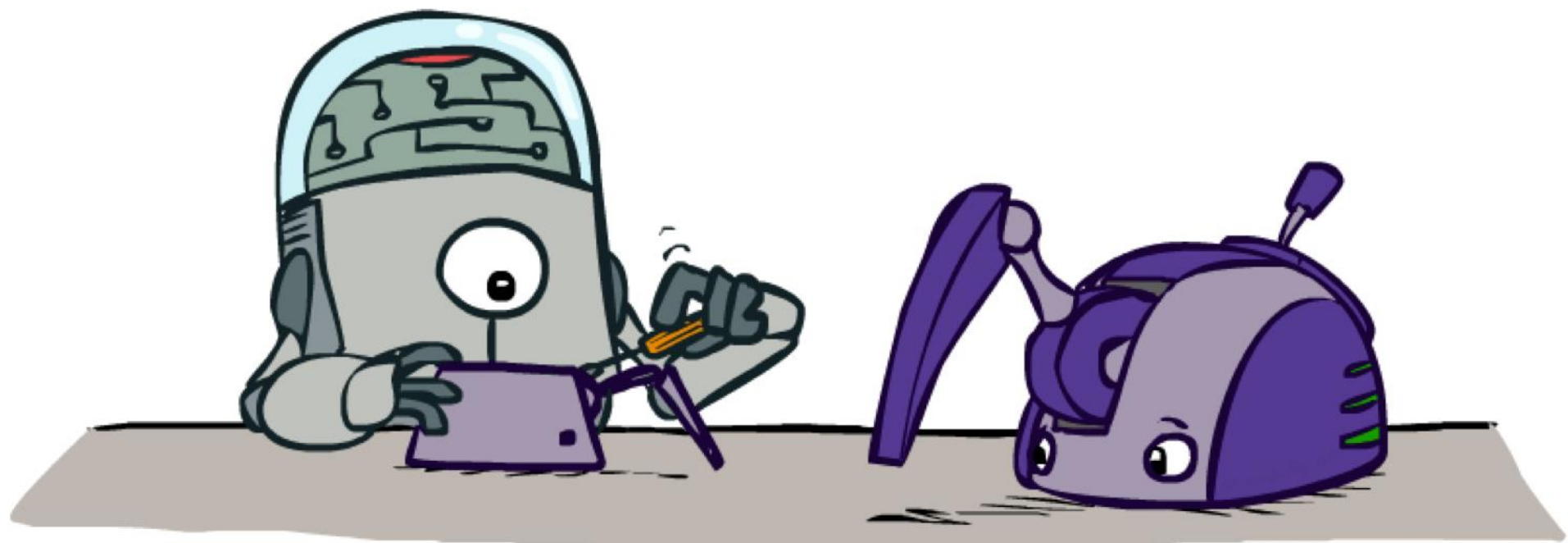


Offline Solution



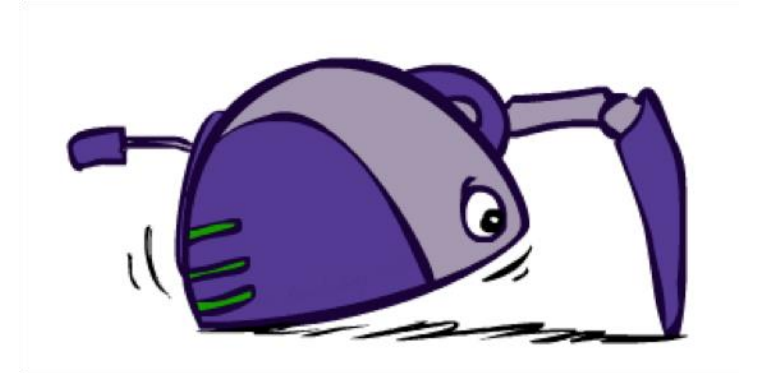
Online Learning

# Model-Based Learning



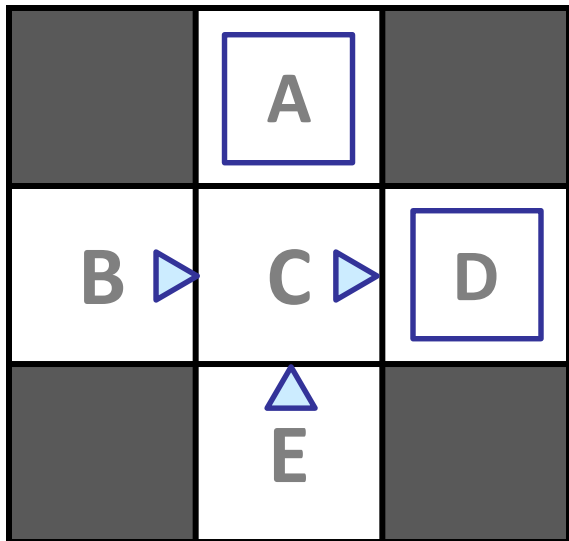
# Model-Based Learning

- **Model-Based Idea:**
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct
- **Step 1: Learn empirical MDP model**
  - Count outcomes  $s'$  for each  $s, a$
  - Normalize to give an estimate of  $\hat{T}(s, a, s')$
  - Discover each  $\hat{R}(s, a, s')$  when we experience  $(s, a, s')$
- **Step 2: Solve the learned MDP**
  - For example, use value iteration, as before



# Example: Model-Based Learning

Input Policy  $\pi$



Assume:  $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 2

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 3

E, north, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

Learned Model

$$\hat{T}(s, a, s')$$

T(B, east, C) = 1.00  
T(C, east, D) = 0.75  
T(C, east, A) = 0.25  
...

$$\hat{R}(s, a, s')$$

R(B, east, C) = -1  
R(C, east, D) = -1  
R(D, exit, x) = +10  
...

# Example: Expected Age

Goal: Compute expected age of CSCI 446 students

Known  $P(A)$

$$E[A] = \sum_a P(a) \cdot a = 0.35 \times 20 + \dots$$

Without  $P(A)$ , instead collect samples  $[a_1, a_2, \dots, a_N]$

Unknown  $P(A)$ : "Model Based"

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$
$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Why does this work? Because eventually you learn the right model.

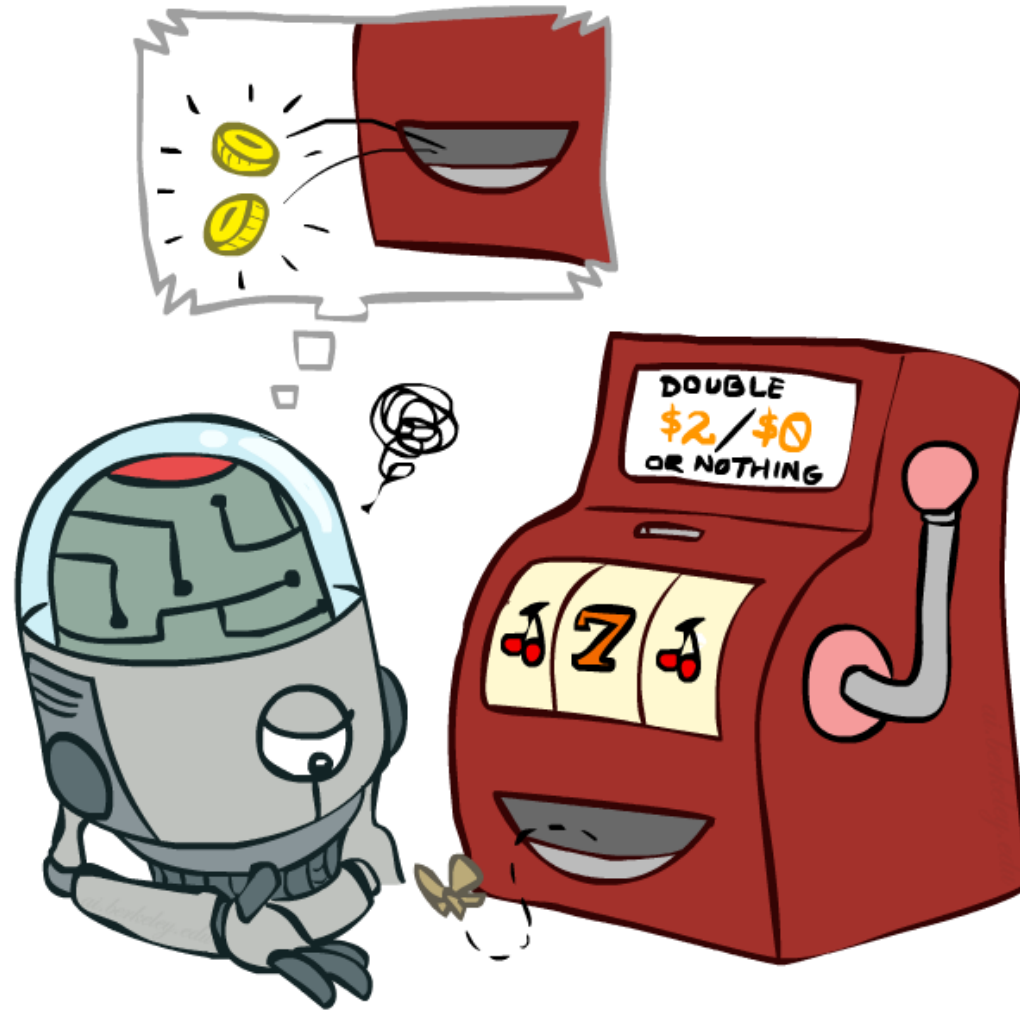
Unknown  $P(A)$ : "Model Free"

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

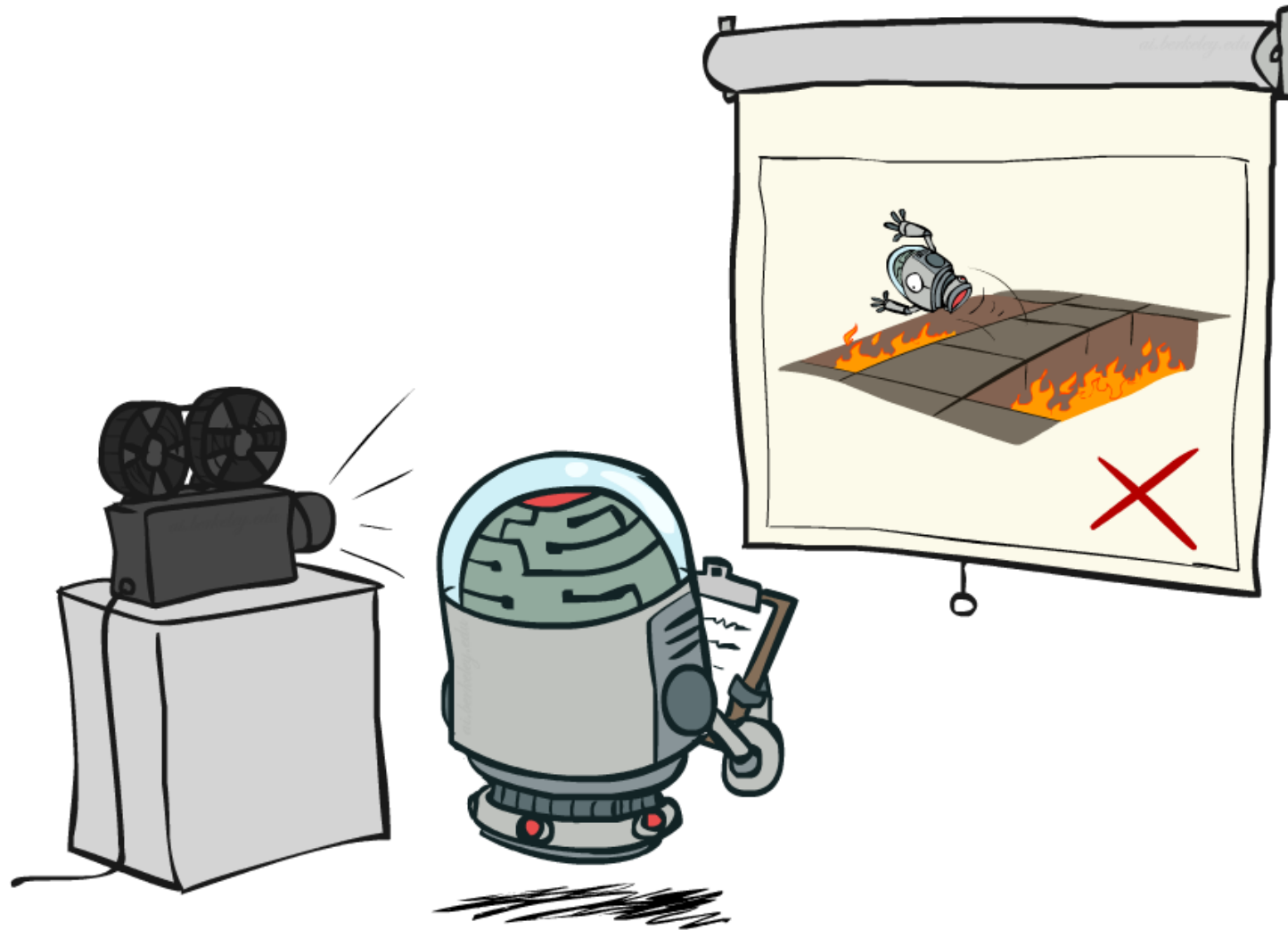
Why does this work? Because samples appear with the right frequencies.



# Model-Free Learning



# Passive Reinforcement Learning



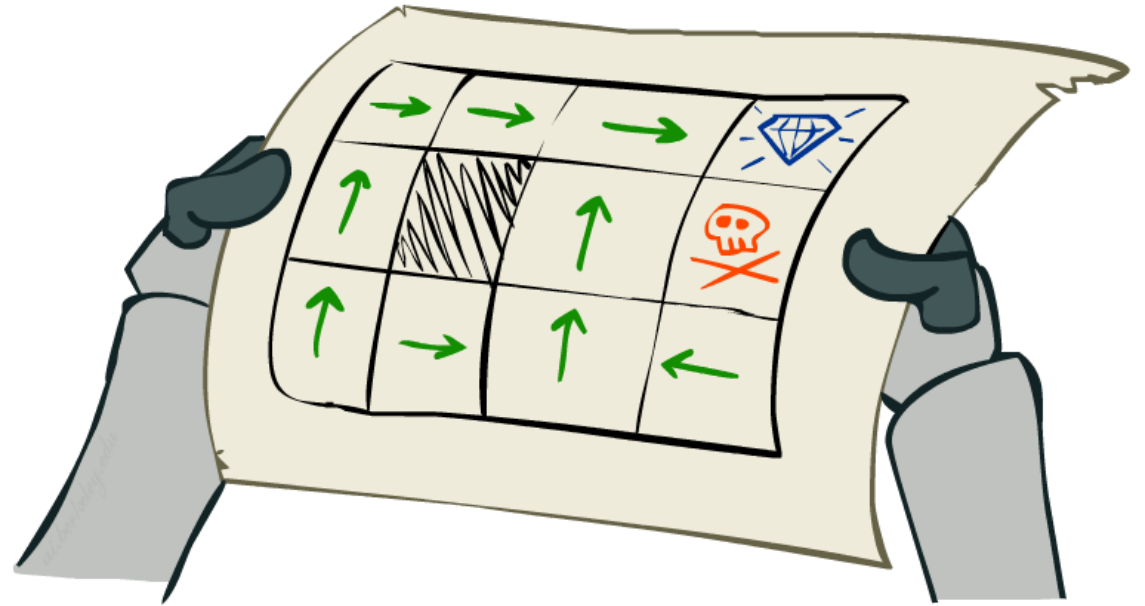
# Passive Reinforcement Learning

- Simplified task: policy evaluation

- Input: a fixed policy  $\pi(s)$
- You don't know the transitions  $T(s,a,s')$
- You don't know the rewards  $R(s,a,s')$
- **Goal: learn the state values**

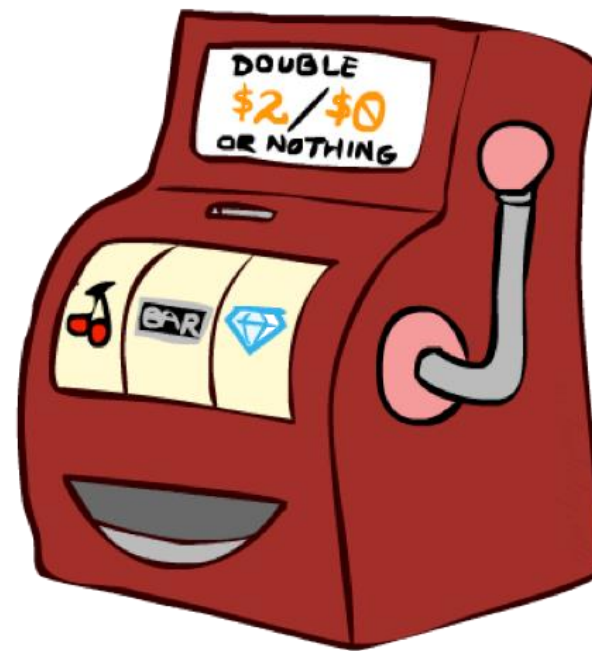
- In this case:

- Learner is “along for the ride”
- No choice about what actions to take
- Just execute the policy and learn from experience
- This is NOT offline planning! You actually take actions in the world.



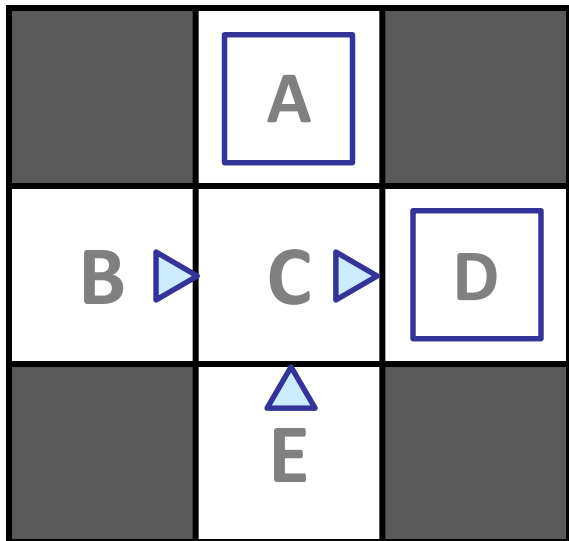
# Direct Evaluation

- Goal: Compute values for each state under  $\pi$
- Idea: Average together observed sample values
  - Act according to  $\pi$
  - Every time you visit a state, write down what the sum of discounted rewards turned out to be
  - Average those samples
- This is called direct evaluation



# Example: Direct Evaluation

Input Policy  $\pi$



Assume:  $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 2

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 3

E, north, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

Output Values

|    |     |     |
|----|-----|-----|
|    | -10 |     |
|    | A   |     |
| +8 | +4  | +10 |
| B  | C   | D   |
|    | -2  |     |
|    | E   |     |

# Problems with Direct Evaluation

- What's good about direct evaluation?
  - It's easy to understand
  - It doesn't require any knowledge of T, R
  - It eventually computes the correct average values, using just sample transitions
- What bad about it?
  - It wastes information about state connections
  - Each state must be learned separately
  - So, it takes a long time to learn

## Output Values

|         |          |          |
|---------|----------|----------|
|         | -10<br>A |          |
| +8<br>B | +4<br>C  | +10<br>D |
|         | -2<br>E  |          |

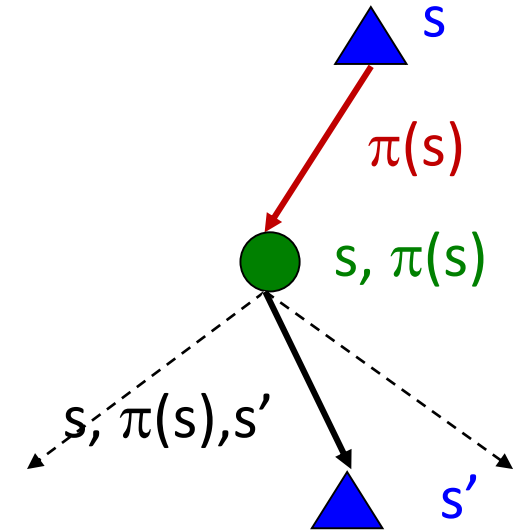
*If B and E both go to C under this policy, how can their values be different?*

# Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate  $V$  for a fixed policy:
  - Each round, replace  $V$  with a one-step-look-ahead layer over  $V$

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



- This approach fully exploited the connections between the states
  - Unfortunately, we need  $T$  and  $R$  to do it!
- Key question: how can we do this update to  $V$  without knowing  $T$  and  $R$ ?
    - In other words, how to we take a weighted average without knowing the weights?

# Sample-Based Policy Evaluation?

- We want to improve our estimate of  $V$  by computing these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

- Idea: Take samples of outcomes  $s'$  (by doing the action!) and average

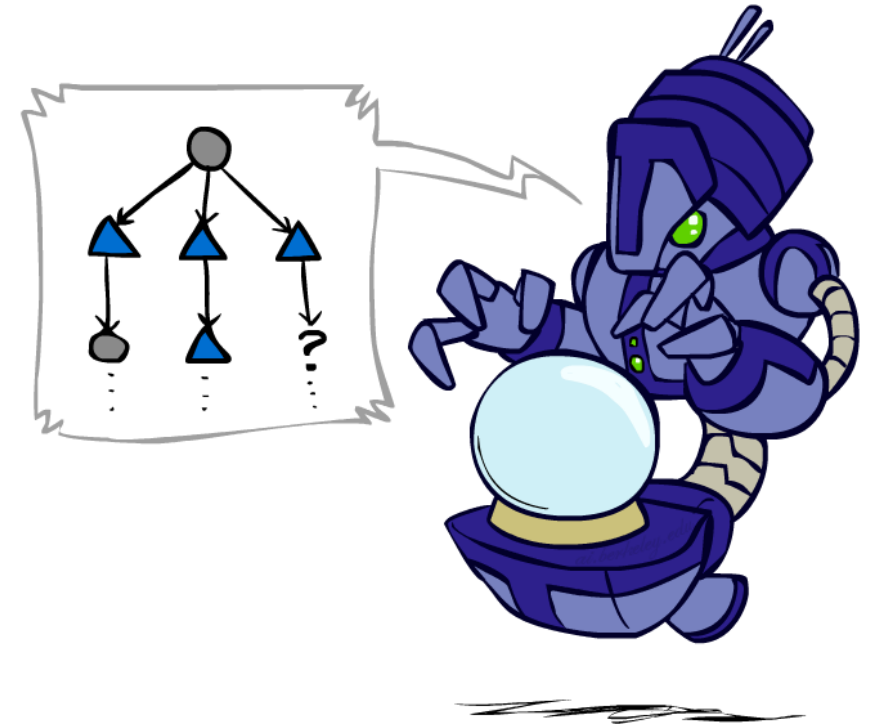
$$\text{sample}_1 = R(s, \pi(s), s'_1) + \gamma V_k^{\pi}(s'_1)$$

$$\text{sample}_2 = R(s, \pi(s), s'_2) + \gamma V_k^{\pi}(s'_2)$$

...

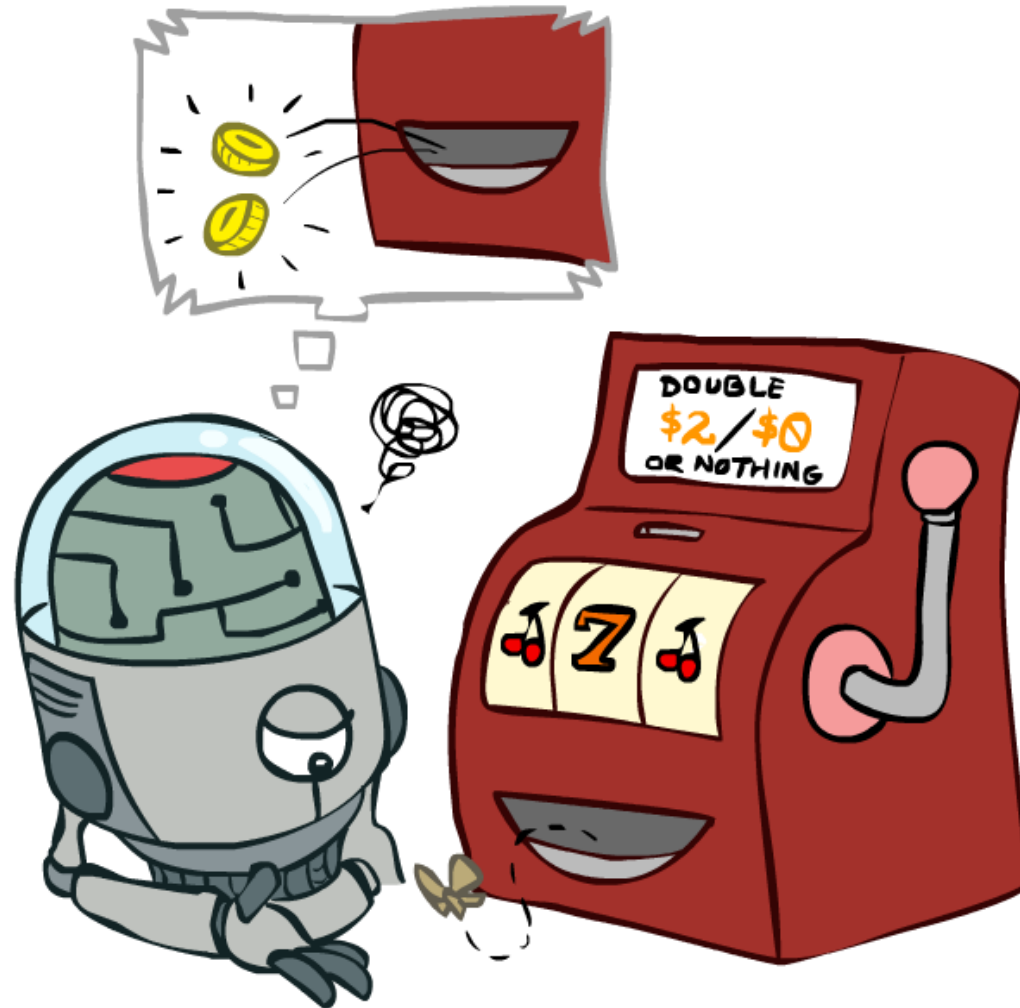
$$\text{sample}_n = R(s, \pi(s), s'_n) + \gamma V_k^{\pi}(s'_n)$$

$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n} \sum_i \text{sample}_i$$



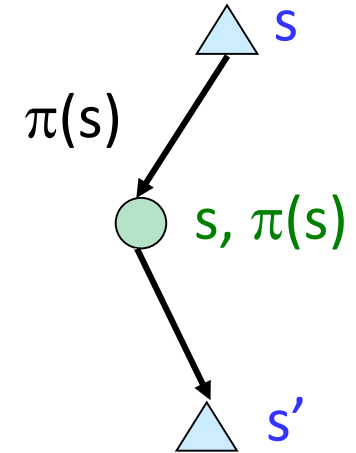


# Temporal Difference Learning



# Temporal Difference Learning

- Big idea: learn from every experience!
  - Update  $V(s)$  each time we experience a transition  $(s, a, s', r)$
  - Likely outcomes  $s'$  will contribute updates more often
- Temporal difference learning of values
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average



Sample of  $V(s)$ :  $sample = R(s, \pi(s), s') + \gamma V^\pi(s')$

Update to  $V(s)$ :  $V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + (\alpha)sample$

Same update:  $V^\pi(s) \leftarrow V^\pi(s) + \alpha(sample - V^\pi(s))$

# Exponential Moving Average

- Exponential moving average

- The running interpolation update:  $\bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$

- Makes recent samples more important:

$$\bar{x}_n = \frac{x_n + (1 - \alpha) \cdot x_{n-1} + (1 - \alpha)^2 \cdot x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + \dots}$$

- Forgets about the past (distant past values were wrong anyway)

- Decreasing learning rate (alpha) can give converging averages

# Example: Temporal Difference Learning

## States

|   |   |   |
|---|---|---|
|   | A |   |
| B | C | D |
|   | E |   |

Assume:  $\gamma = 1$ ,  $\alpha = 1/2$

## Observed Transitions

B, east, C, -2

C, east, D, -2

|   |   |   |
|---|---|---|
|   | 0 |   |
| 0 | 0 | 8 |
|   | 0 |   |

|    |   |   |
|----|---|---|
|    | 0 |   |
| -1 | 0 | 8 |
|    | 0 |   |

|    |   |   |
|----|---|---|
|    | 0 |   |
| -1 | 3 | 8 |
|    | 0 |   |

$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

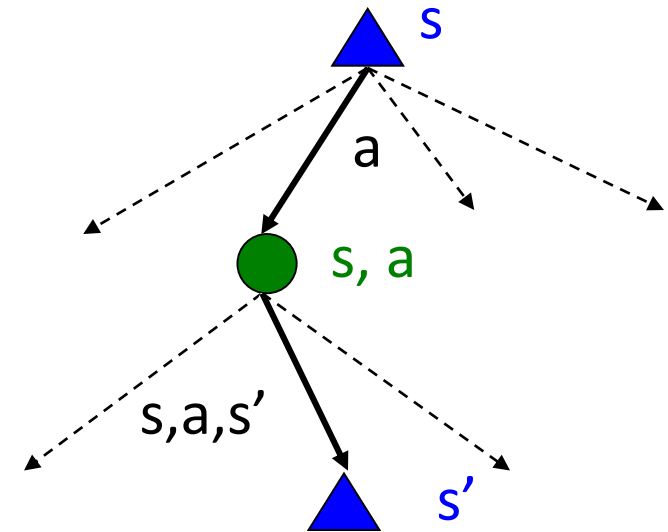
# Problems with TD Value Learning

- TD value learning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

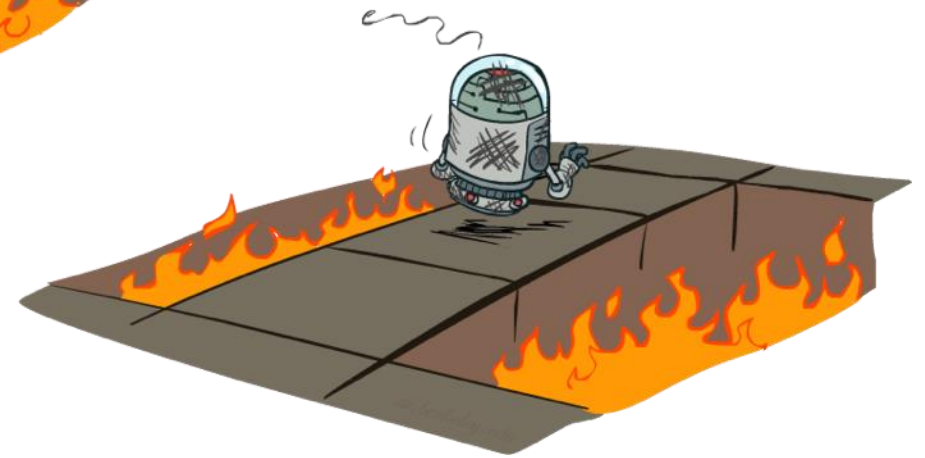
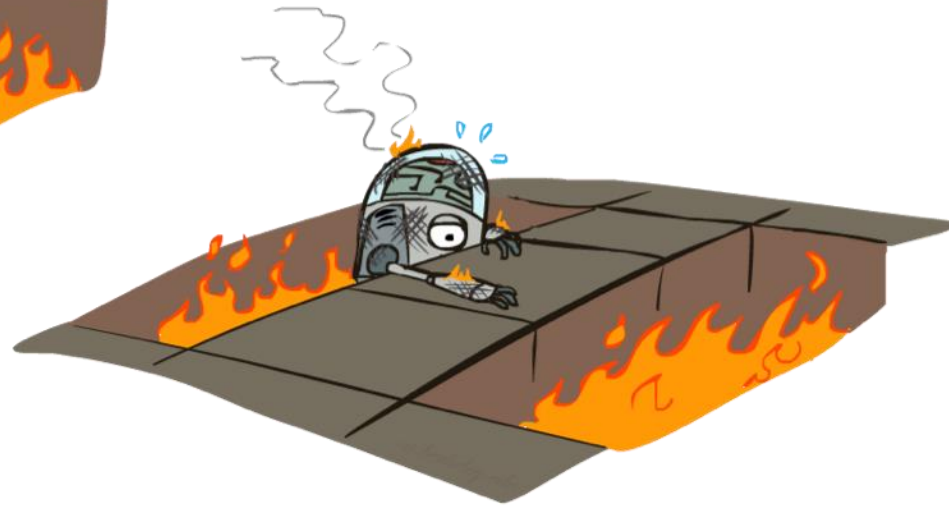
$$\pi(s) = \arg \max_a Q(s, a)$$

$$Q(s, a) = \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V(s')]$$

- Idea: learn Q-values, not values
- Makes action selection model-free too!

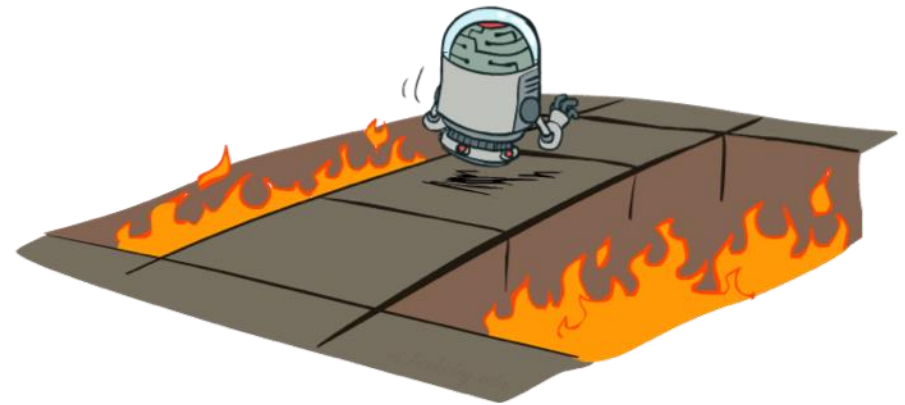


# Active Reinforcement Learning



# Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
  - You don't know the transitions  $T(s,a,s')$
  - You don't know the rewards  $R(s,a,s')$
  - You choose the actions now
  - Goal: learn the optimal policy / values
- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens...



# Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values

- Start with  $V_0(s) = 0$ , which we know is right
- Given  $V_k$ , calculate the depth  $k+1$  values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma V_k(s') \right]$$

- But Q-values are more useful, so compute them instead

- Start with  $Q_0(s,a) = 0$ , which we know is right
- Given  $Q_k$ , calculate the depth  $k+1$  q-values for all q-states:

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$



# Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

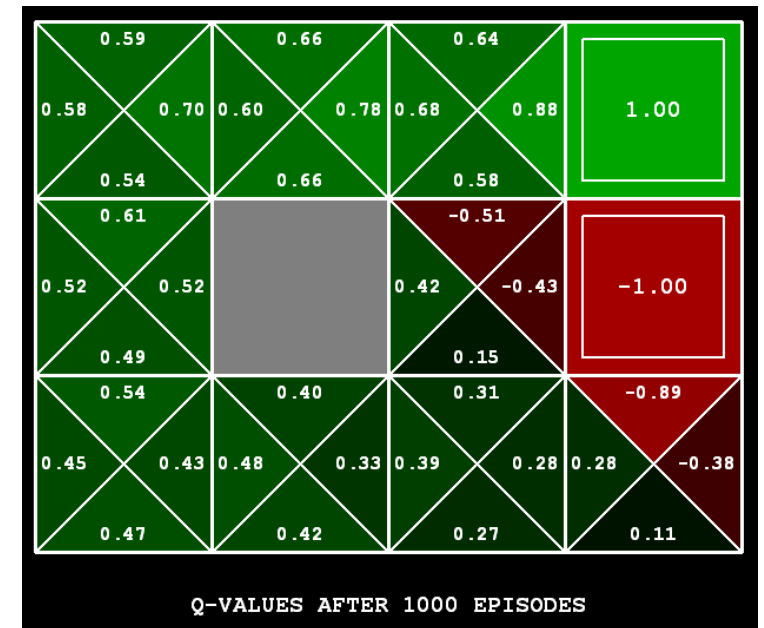
- Learn  $Q(s,a)$  values as you go

- Receive a sample  $(s,a,s',r)$
- Consider your old estimate:  $Q(s, a)$
- Consider your new sample estimate:

$$sample = R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

- Incorporate the new estimate into a running average:

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) [sample]$$

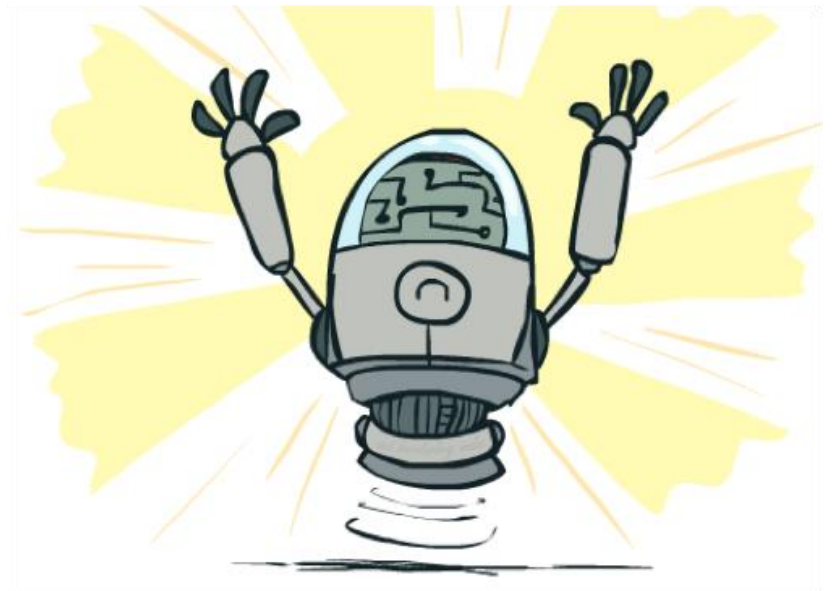


[Demo: Q-learning – gridworld (L10D2)]

[Demo: Q-learning – crawler (L10D3)]

# Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!
- This is called **off-policy learning**
- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - ... but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions (!)



# Today

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  - Temporal Difference Learning
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