

CSCI 446: Artificial Intelligence

Markov Decision Processes



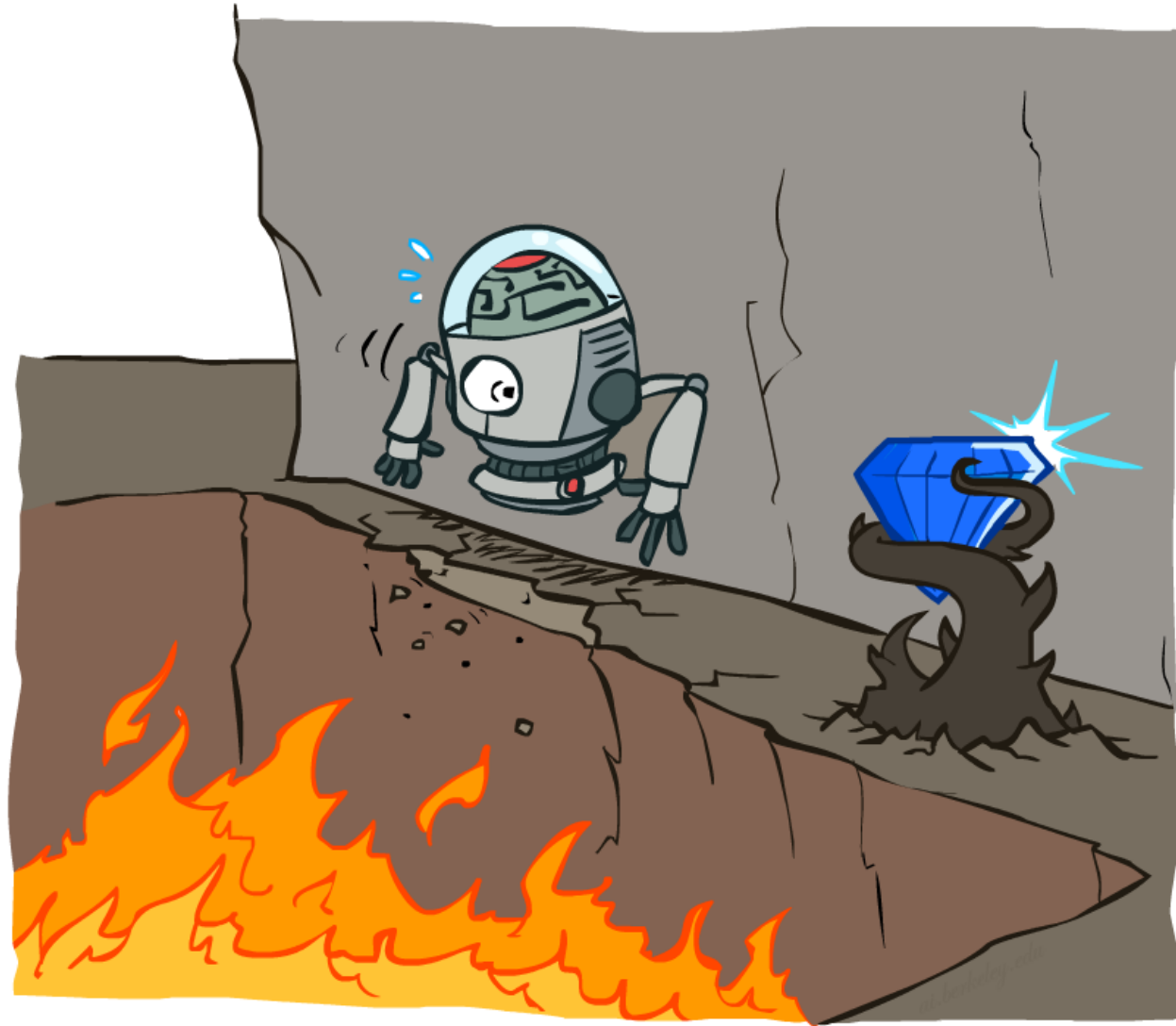
Instructor: Michele Van Dyne

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at <http://ai.berkeley.edu>.]

Today

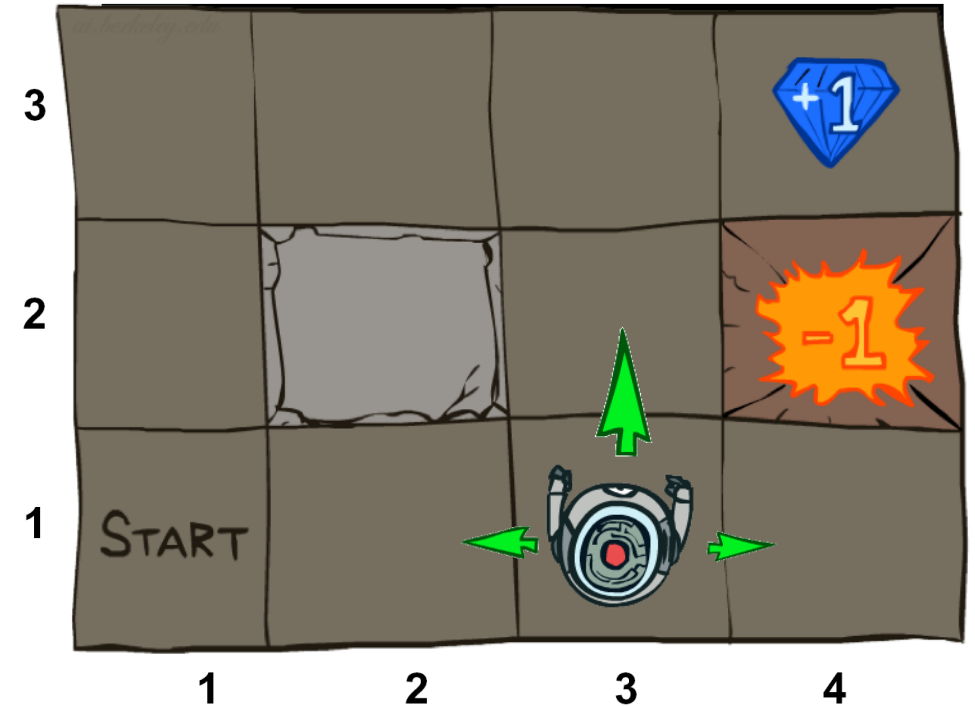
- Non-deterministic Search
- Utilities of Sequences

Non-Deterministic Search



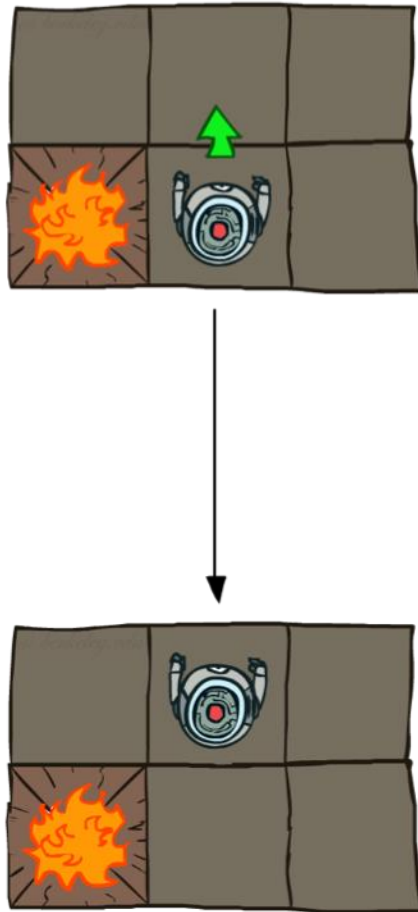
Example: Grid World

- A maze-like problem
 - The agent lives in a grid
 - Walls block the agent's path
- Noisy movement: actions do not always go as planned
 - 80% of the time, the action North takes the agent North (if there is no wall there)
 - 10% of the time, North takes the agent West; 10% East
 - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
 - Small "living" reward each step (can be negative)
 - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards

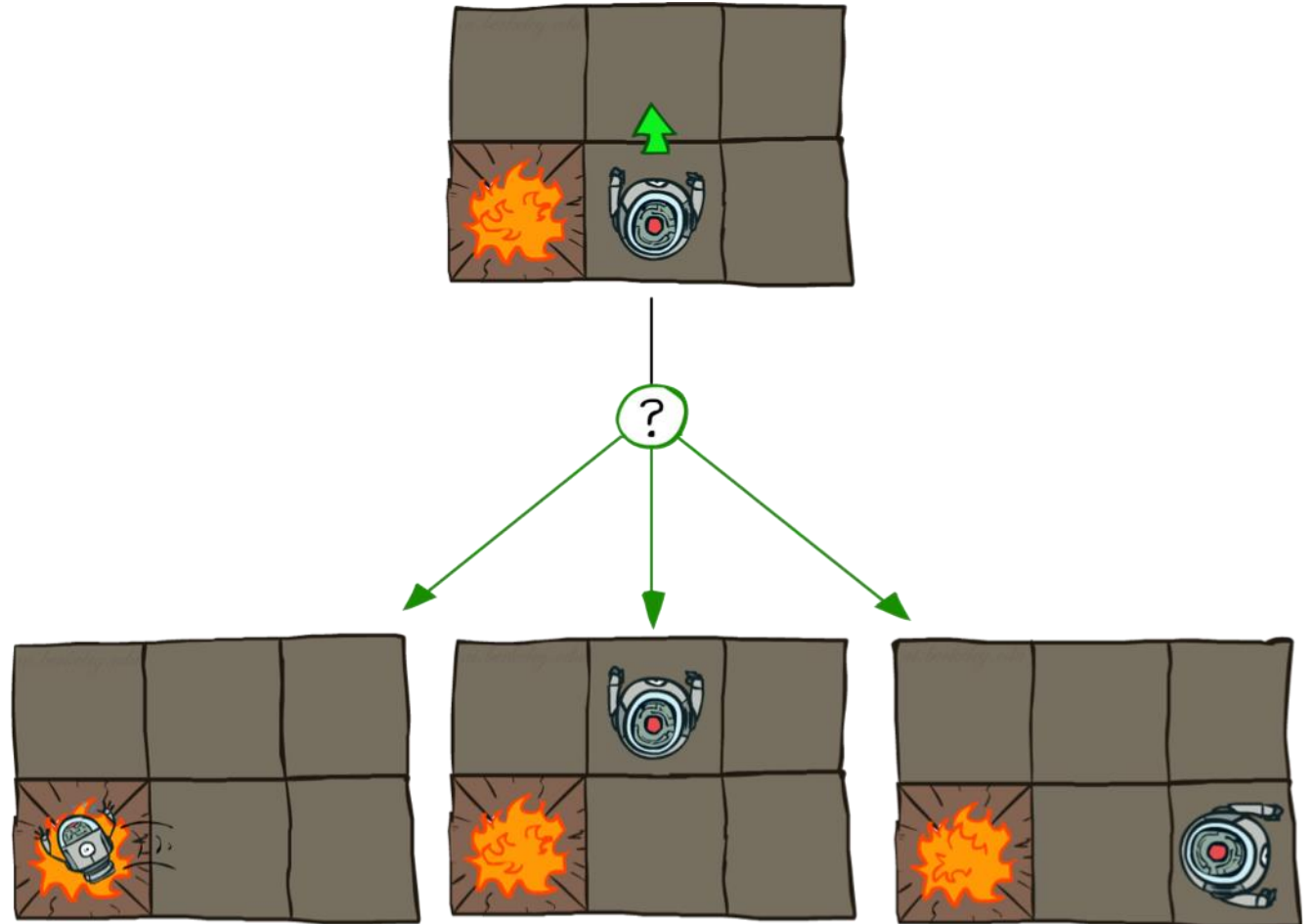


Grid World Actions

Deterministic Grid World

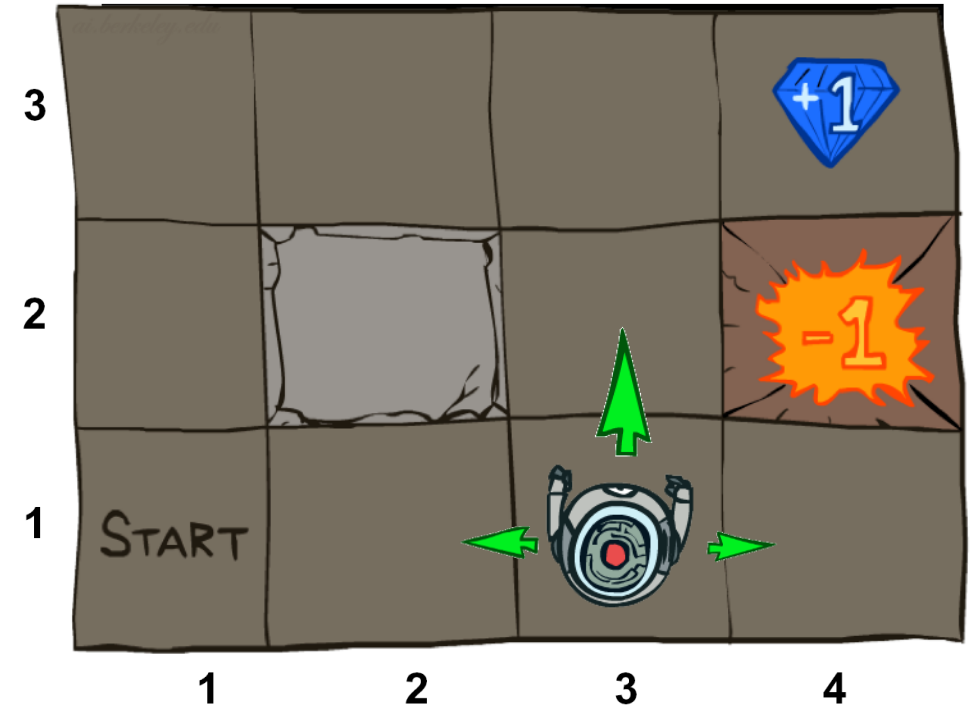


Stochastic Grid World



Markov Decision Processes

- An MDP is defined by:
 - A **set of states** $s \in S$
 - A **set of actions** $a \in A$
 - A **transition function** $T(s, a, s')$
 - Probability that a from s leads to s' , i.e., $P(s' | s, a)$
 - Also called the model or the dynamics
 - A **reward function** $R(s, a, s')$
 - Sometimes just $R(s)$ or $R(s')$
 - A **start state**
 - Maybe a **terminal state**
- MDPs are **non-deterministic search problems**
 - One way to solve them is with expectimax search
 - We'll have a new tool soon



What is Markov about MDPs?

- “Markov” generally means that given the present state, the future and the past are independent
- For Markov decision processes, “Markov” means action outcomes depend only on the current state

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \dots, S_0 = s_0)$$

=

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

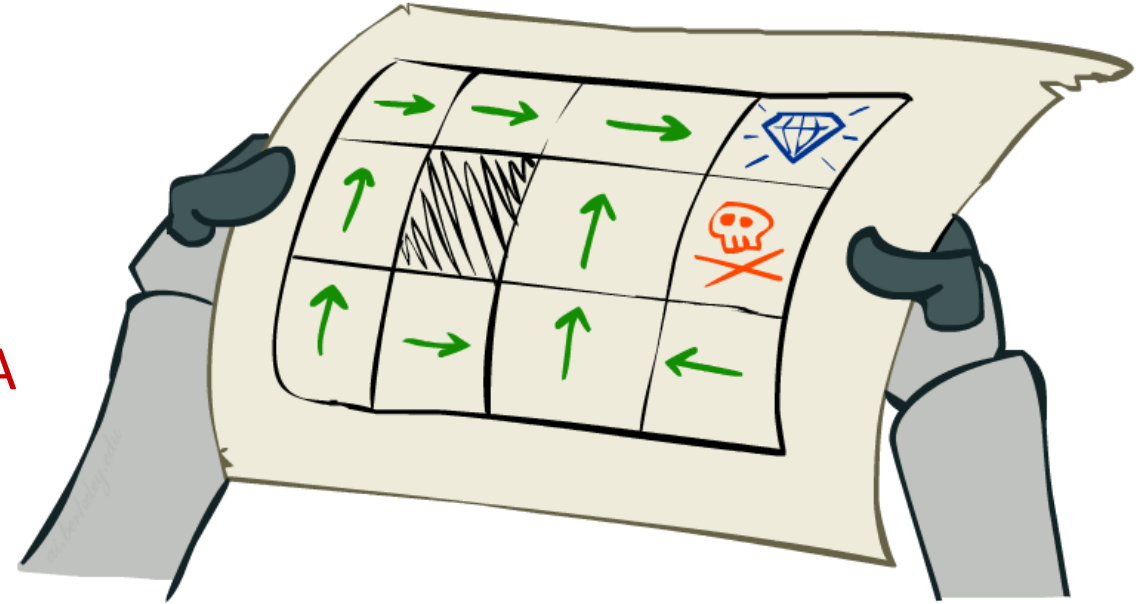
- This is just like search, where the successor function could only depend on the current state (not the history)



Andrey Markov
(1856-1922)

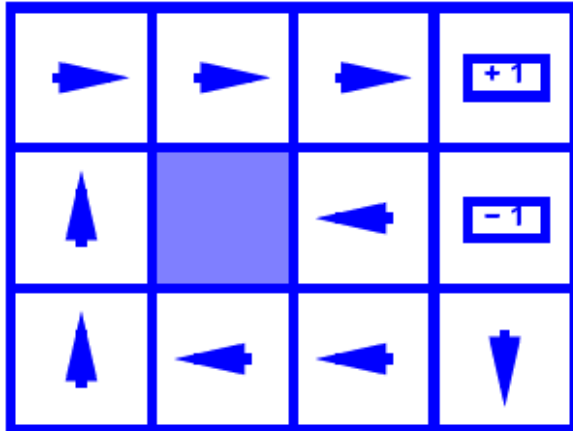
Policies

- In deterministic single-agent search problems, we wanted an optimal **plan**, or sequence of actions, from start to a goal
- For MDPs, we want an optimal **policy** $\pi^*: S \rightarrow A$
 - A policy π gives an action for each state
 - An optimal policy is one that maximizes expected utility if followed
 - An explicit policy defines a reflex agent
- Expectimax didn't compute entire policies
 - It computed the action for a single state only

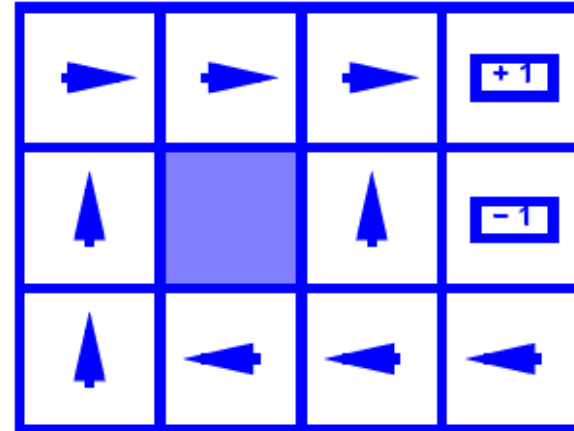


Optimal policy when $R(s, a, s') = -0.03$
for all non-terminals s

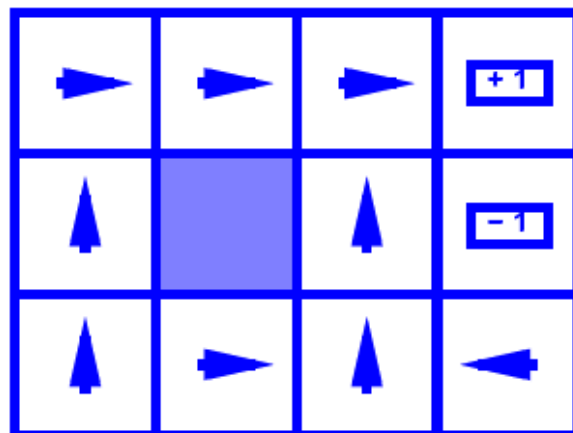
Optimal Policies



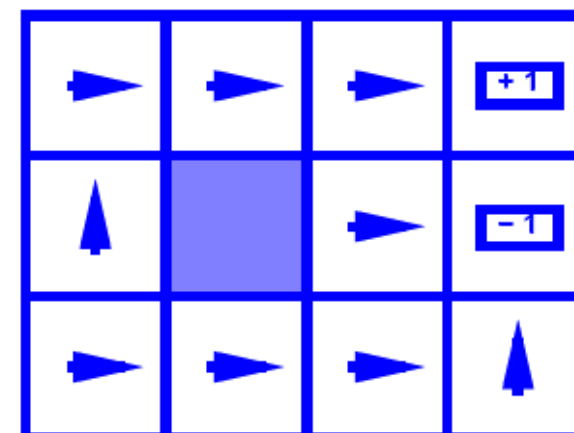
$$R(s) = -0.01$$



$$R(s) = -0.03$$

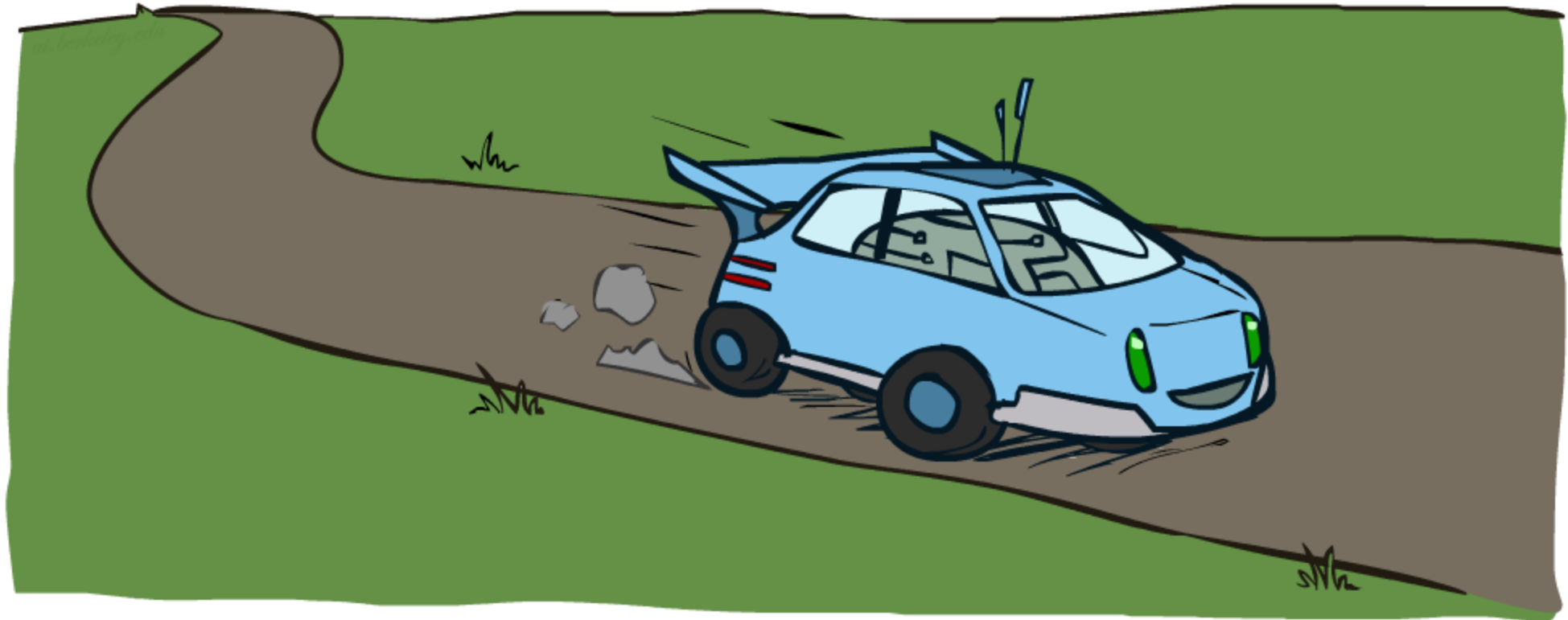


$$R(s) = -0.4$$



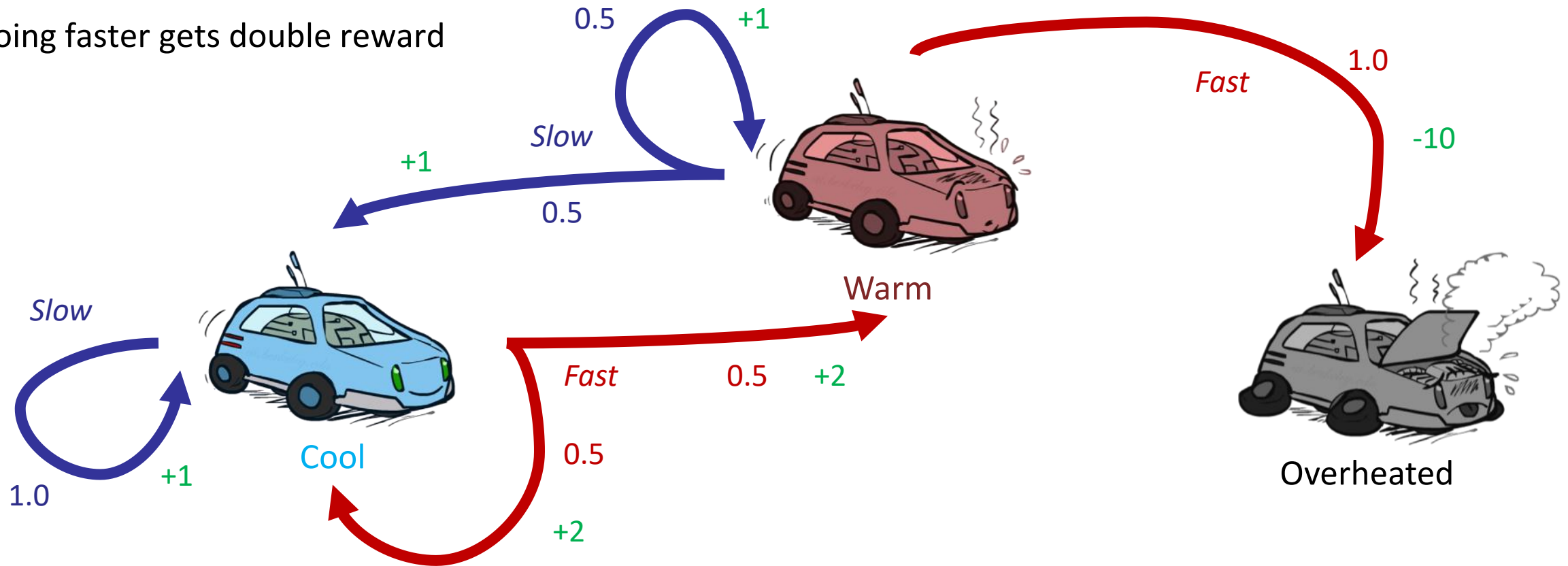
$$R(s) = -2.0$$

Example: Racing

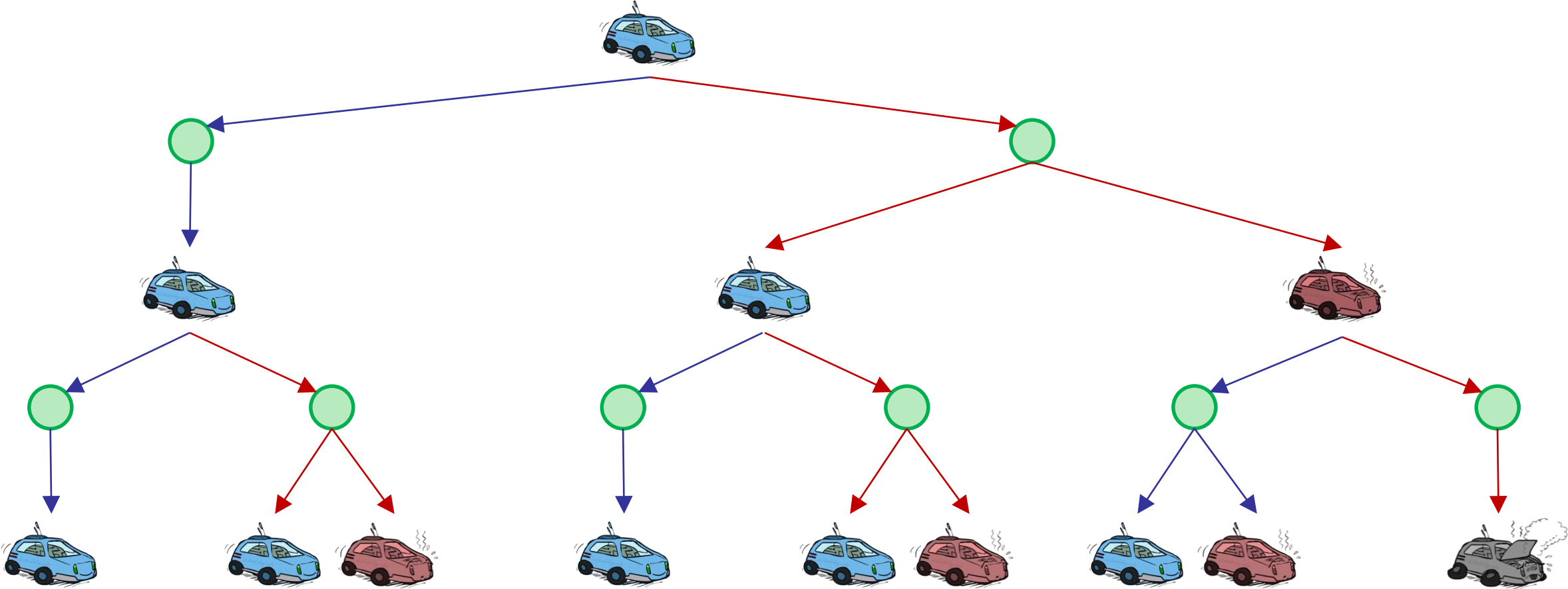


Example: Racing

- A robot car wants to travel far, quickly
- Three states: **Cool**, **Warm**, Overheated
- Two actions: **Slow**, **Fast**
- Going faster gets double reward

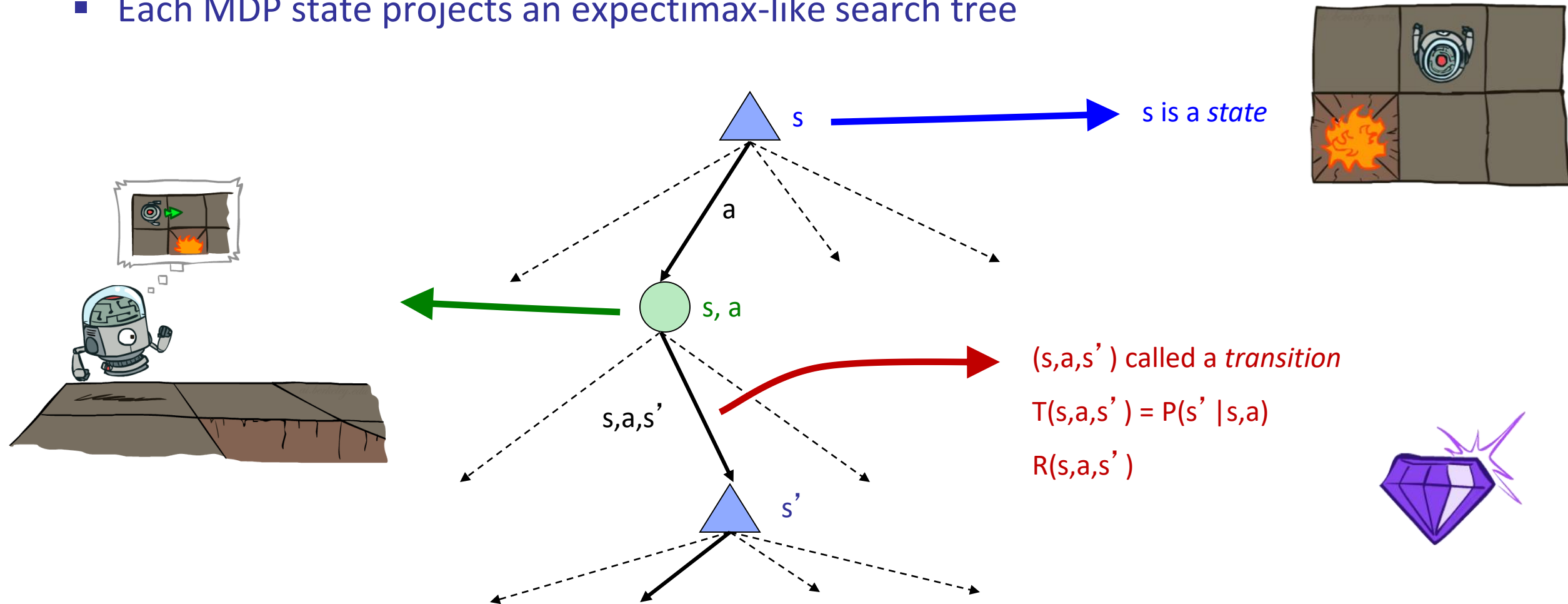


Racing Search Tree

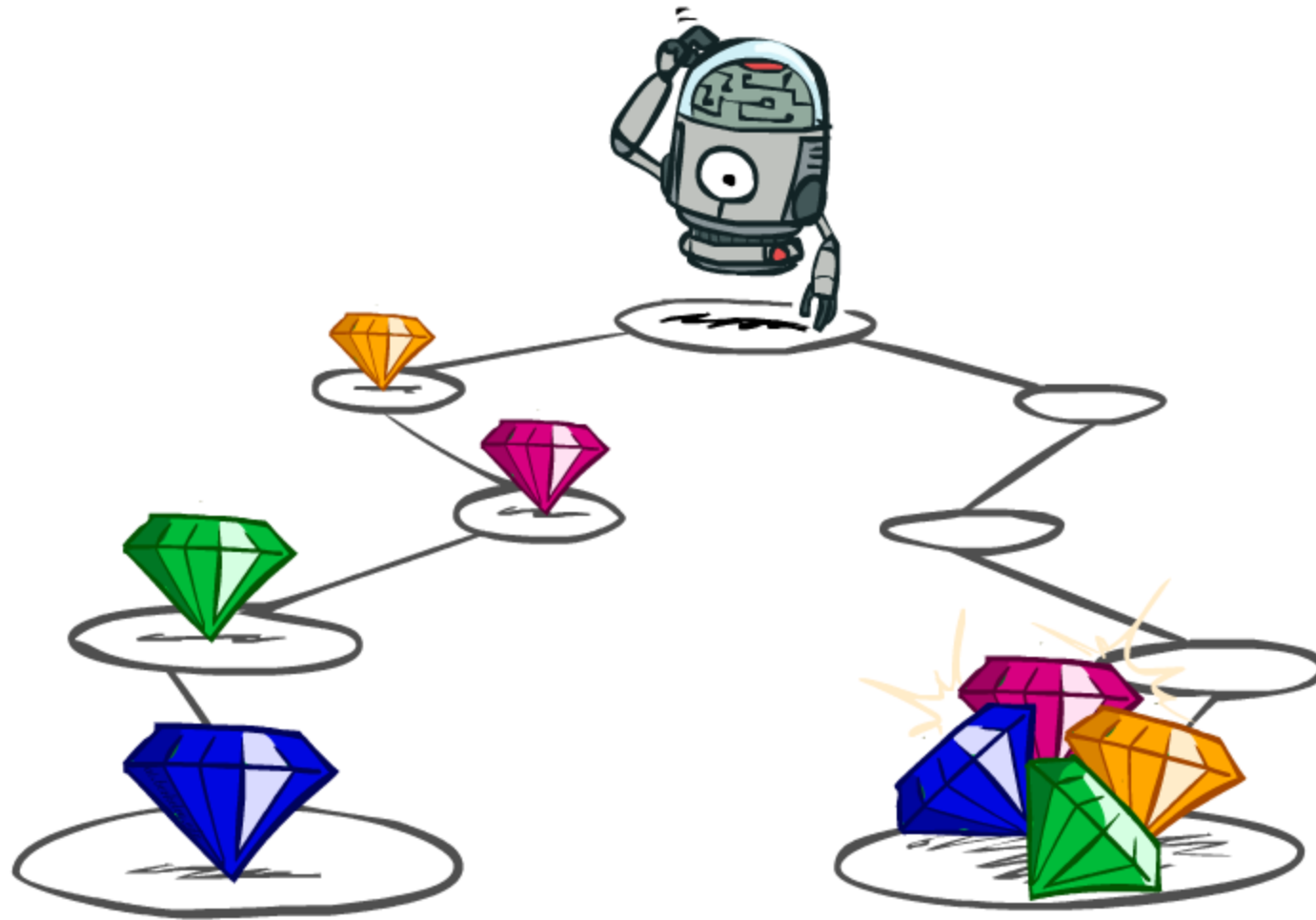


MDP Search Trees

- Each MDP state projects an expectimax-like search tree

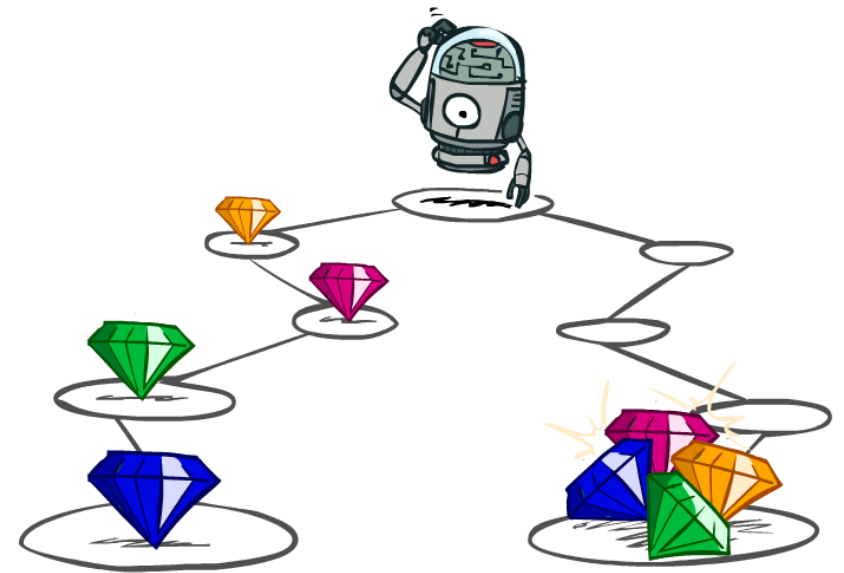


Utilities of Sequences



Utilities of Sequences

- What preferences should an agent have over reward sequences?
- More or less? $[1, 2, 2]$ or $[2, 3, 4]$
- Now or later? $[0, 0, 1]$ or $[1, 0, 0]$



Discounting

- It's reasonable to maximize the sum of rewards
- It's also reasonable to prefer rewards now to rewards later
- One solution: values of rewards decay exponentially



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Worth Now



γ

Worth Next Step



γ^2

Worth In Two Steps

Discounting

- How to discount?

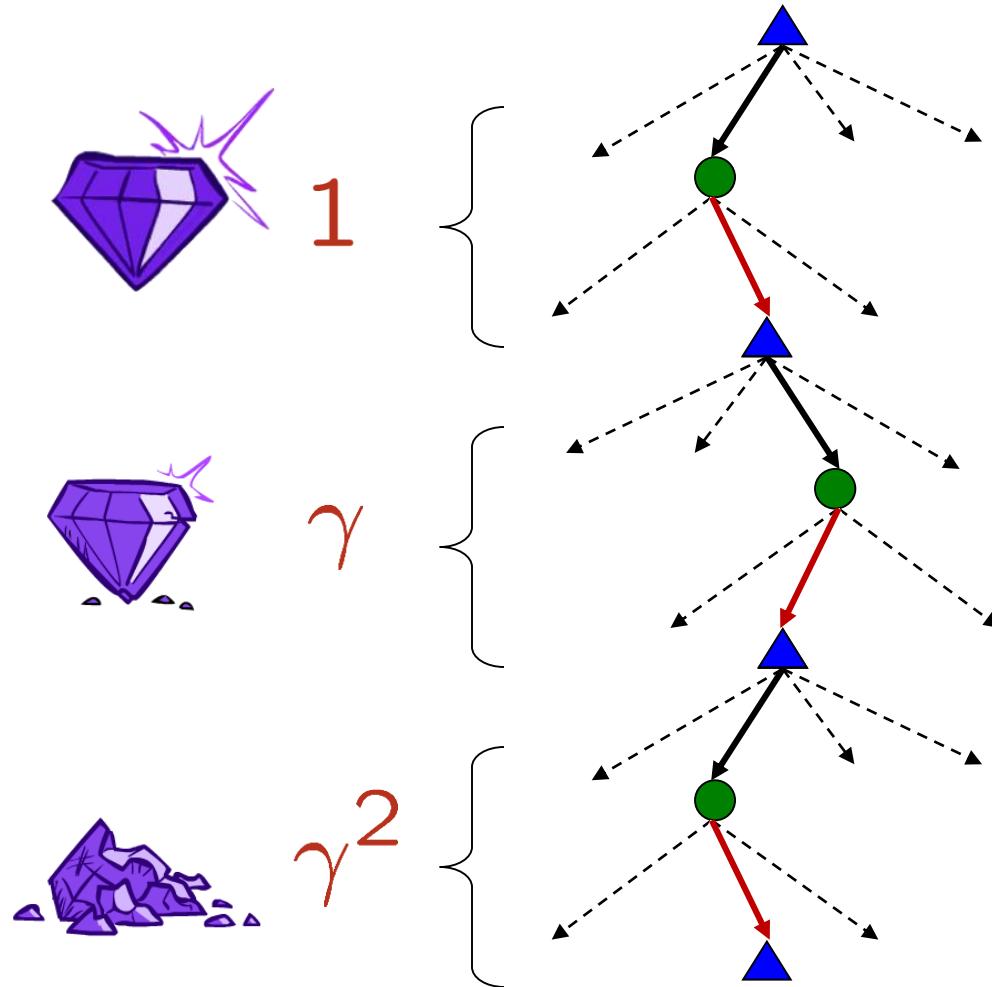
- Each time we descend a level, we multiply in the discount once

- Why discount?

- Sooner rewards probably do have higher utility than later rewards
- Also helps our algorithms converge

- Example: discount of 0.5

- $U([1,2,3]) = 1*1 + 0.5*2 + 0.25*3$
- $U([1,2,3]) < U([3,2,1])$



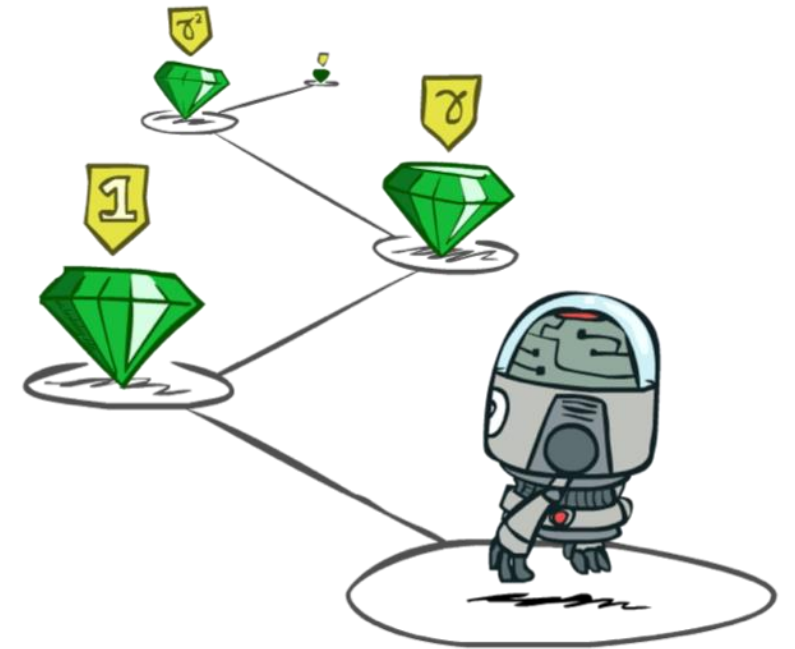
Stationary Preferences

- Theorem: if we assume **stationary preferences**:

$$[a_1, a_2, \dots] \succ [b_1, b_2, \dots]$$



$$[r, a_1, a_2, \dots] \succ [r, b_1, b_2, \dots]$$



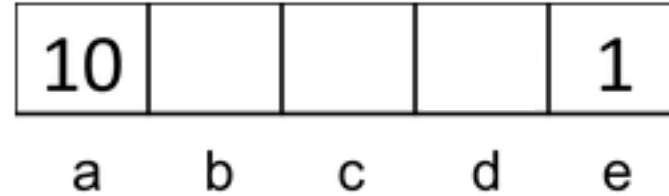
- Then: there are only two ways to define utilities

- Additive utility: $U([r_0, r_1, r_2, \dots]) = r_0 + r_1 + r_2 + \dots$

- Discounted utility: $U([r_0, r_1, r_2, \dots]) = r_0 + \gamma r_1 + \gamma^2 r_2 \dots$

Quiz: Discounting

- Given:



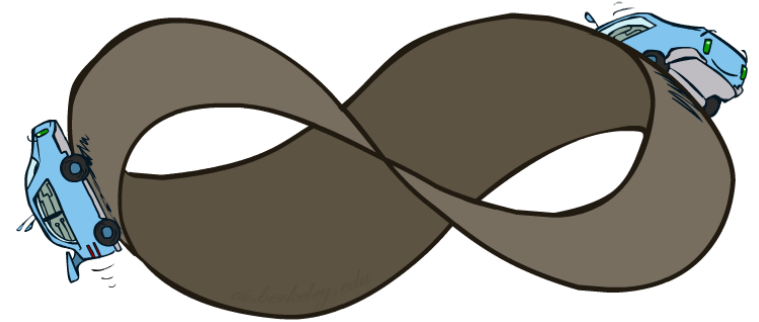
- Actions: East, West, and Exit (only available in exit states a, e)
 - Transitions: deterministic
-
- Quiz 1: For $\gamma = 1$, what is the optimal policy?
- | | | | | |
|----|--|--|--|---|
| 10 | | | | 1 |
|----|--|--|--|---|
- Quiz 2: For $\gamma = 0.1$, what is the optimal policy?
- | | | | | |
|----|--|--|--|---|
| 10 | | | | 1 |
|----|--|--|--|---|
- Quiz 3: For which γ are West and East equally good when in state d?

Infinite Utilities?!

- Problem: What if the game lasts forever? Do we get infinite rewards?

- Solutions:

- Finite horizon: (similar to depth-limited search)
 - Terminate episodes after a fixed T steps (e.g. life)
 - Gives nonstationary policies (π depends on time left)



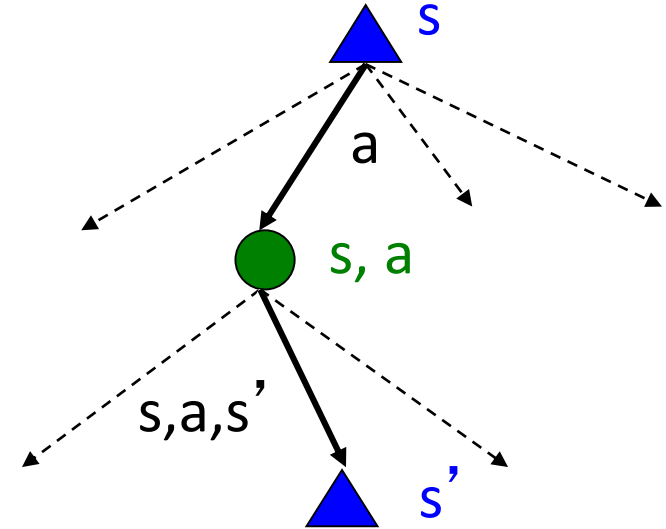
- Discounting: use $0 < \gamma < 1$

$$U([r_0, \dots, r_\infty]) = \sum_{t=0}^{\infty} \gamma^t r_t \leq R_{\max}/(1 - \gamma)$$

- Smaller γ means smaller “horizon” – shorter term focus
- Absorbing state: guarantee that for every policy, a terminal state will eventually be reached (like “overheated” for racing)

Recap: Defining MDPs

- Markov decision processes:
 - Set of states S
 - Start state s_0
 - Set of actions A
 - Transitions $P(s' | s, a)$ (or $T(s, a, s')$)
 - Rewards $R(s, a, s')$ (and discount γ)
- MDP quantities so far:
 - Policy = Choice of action for each state
 - Utility = sum of (discounted) rewards



Today

- Non-deterministic Search
- Utilities of Sequences

