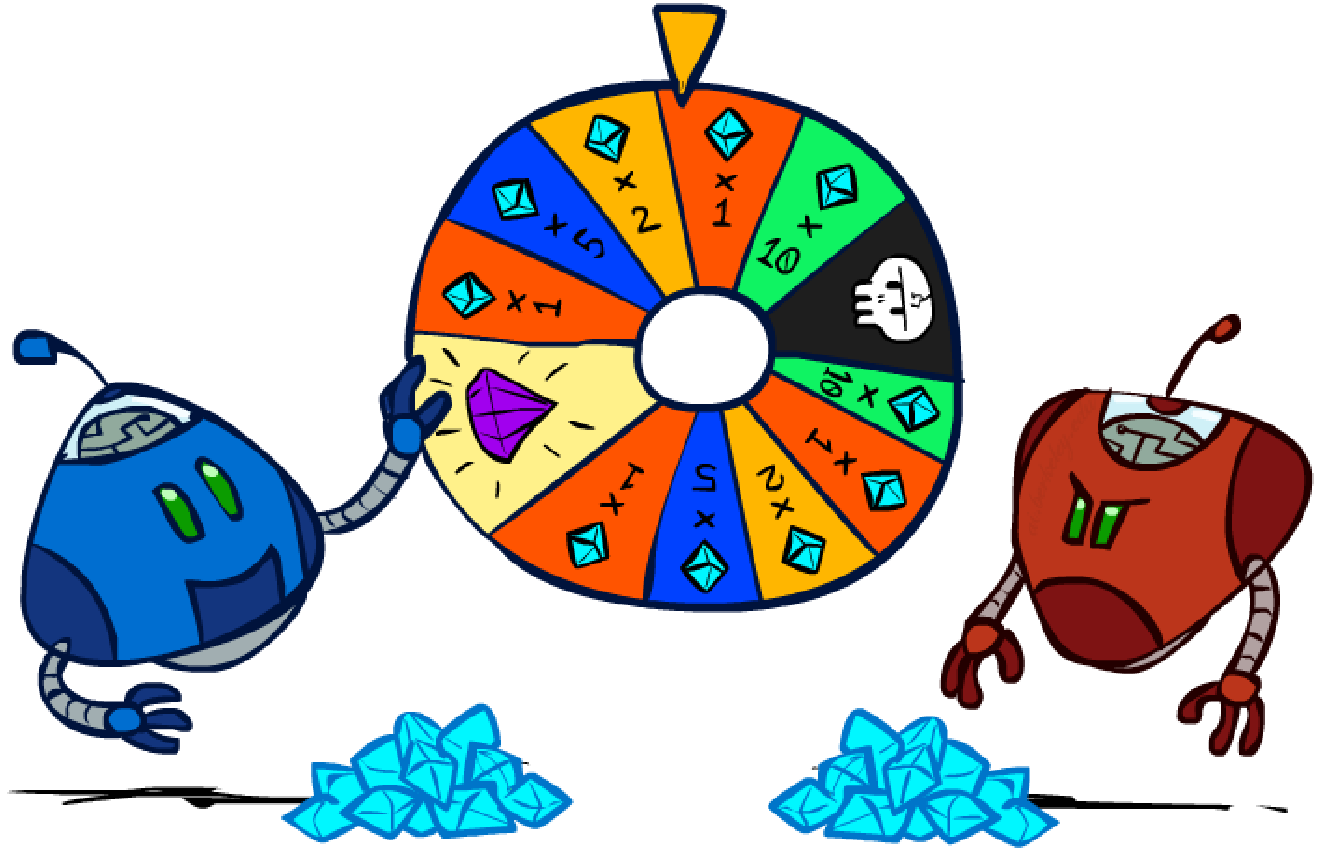
CSCI 446: Artificial Intelligence

Uncertainty and Utilities



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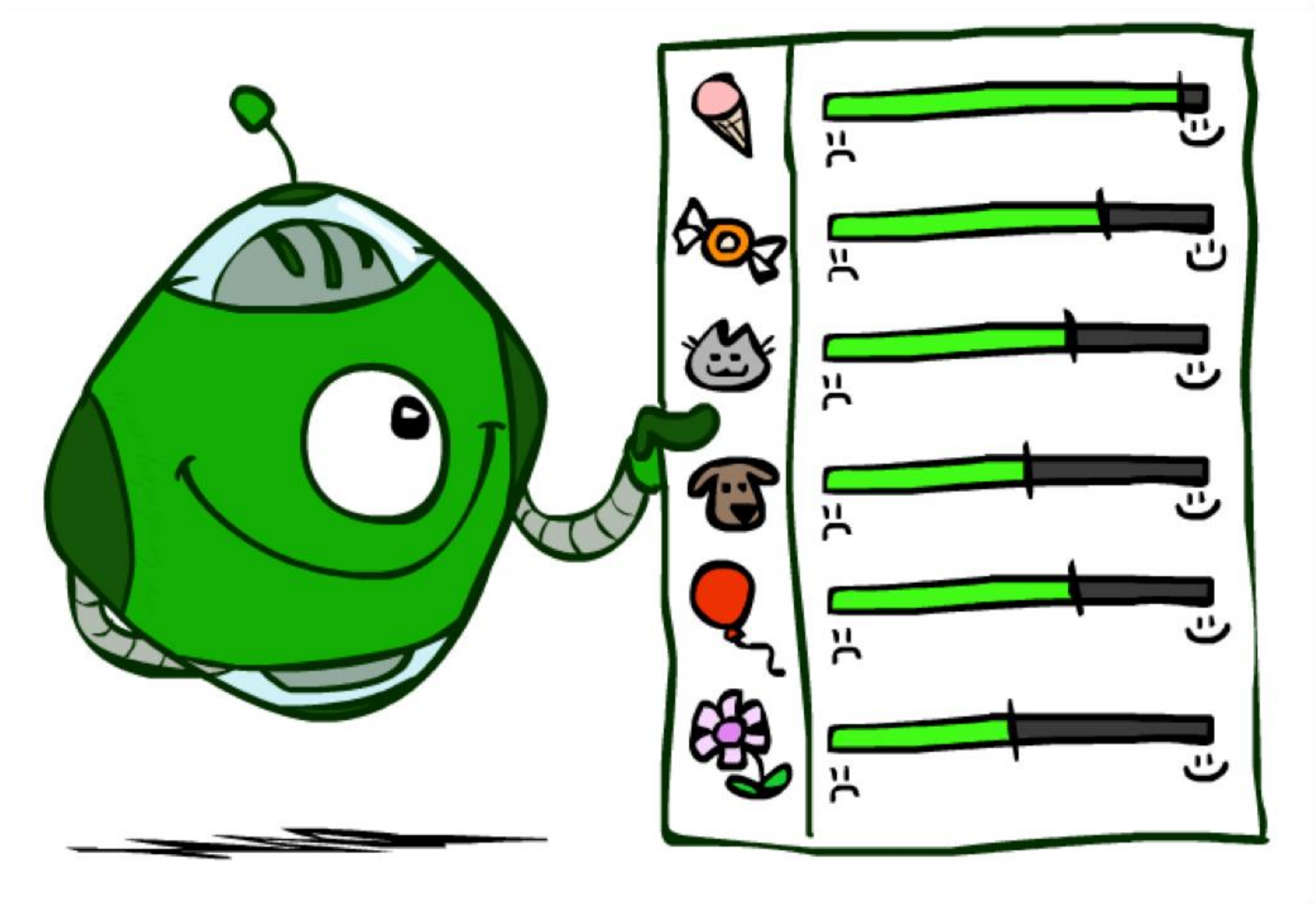
[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]

Today

- Rationality
- Human Utilities

Utilities

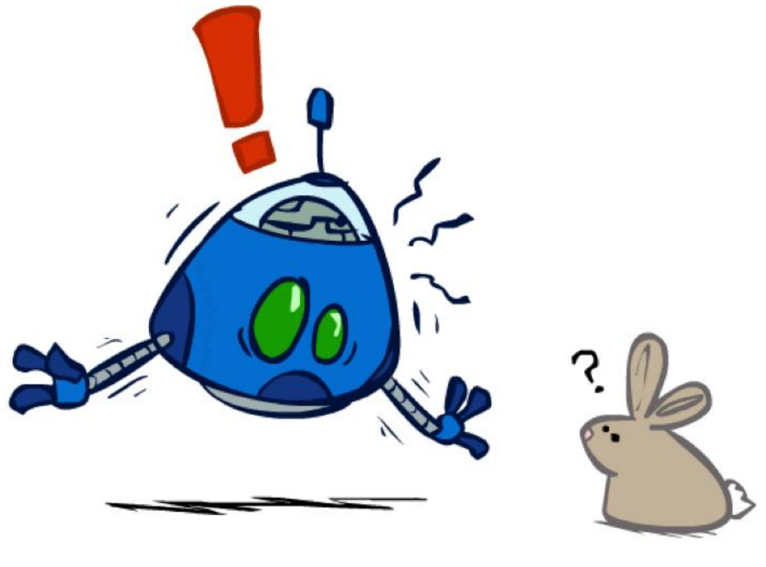


Maximum Expected Utility

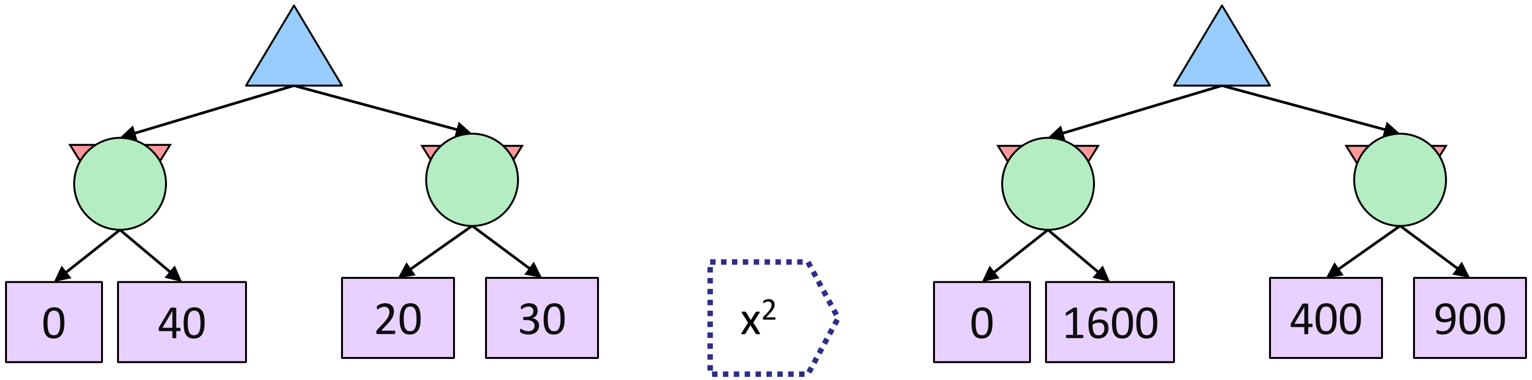
- Why should we average utilities? Why not minimax?

- Principle of maximum expected utility:
  - A rational agent should chose the action that maximizes its expected utility, given its knowledge

- Questions:
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - How do we know that averaging even makes sense?
  - What if our behavior (preferences) can't be described by utilities?



What Utilities to Use?



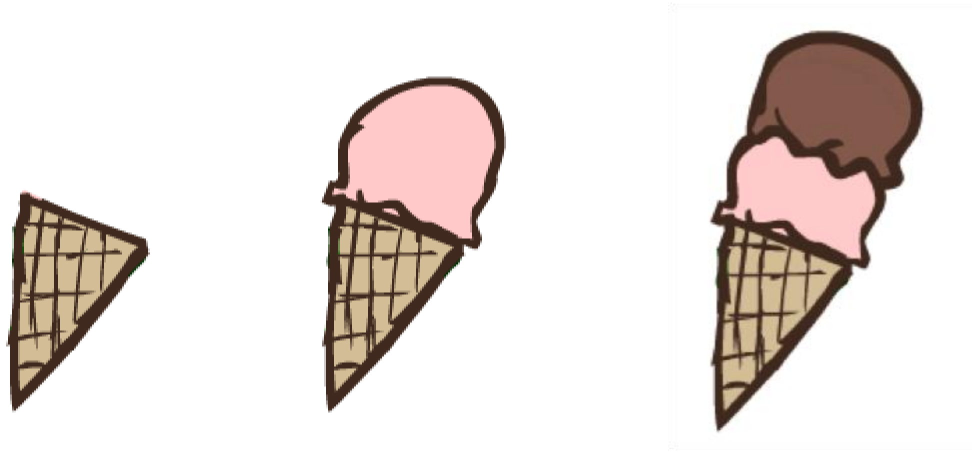
- For worst-case minimax reasoning, terminal function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- For average-case expectimax reasoning, we need *magnitudes* to be meaningful

Utilities

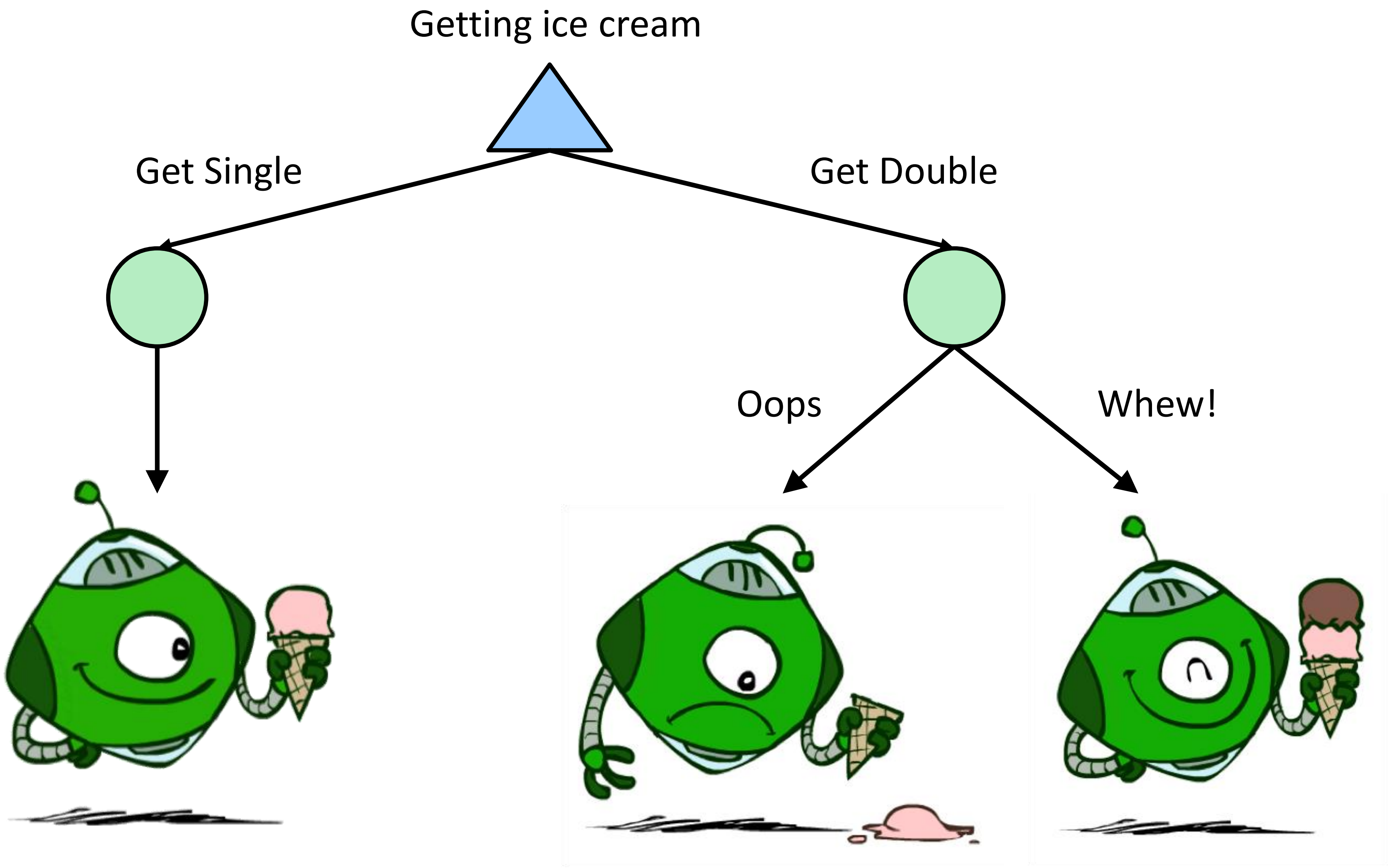
- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences

- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function

- We hard-wire utilities and let behaviors emerge
  - Why don't we let agents pick utilities?
  - Why don't we prescribe behaviors?



Utilities: Uncertain Outcomes



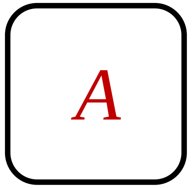
Preferences

- An agent must have preferences among:
  - Prizes: A, B, etc.
  - Lotteries: situations with uncertain prizes

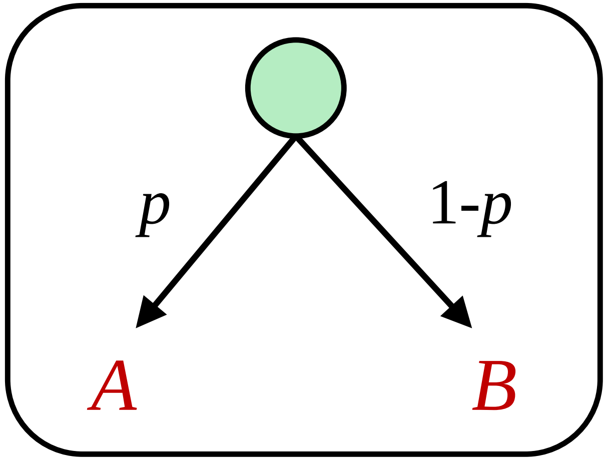
$$L = [p, A;\ (1-p), B]$$

- Notation:
  - Preference: $A \succ B$
  - Indifference: $A \sim B$

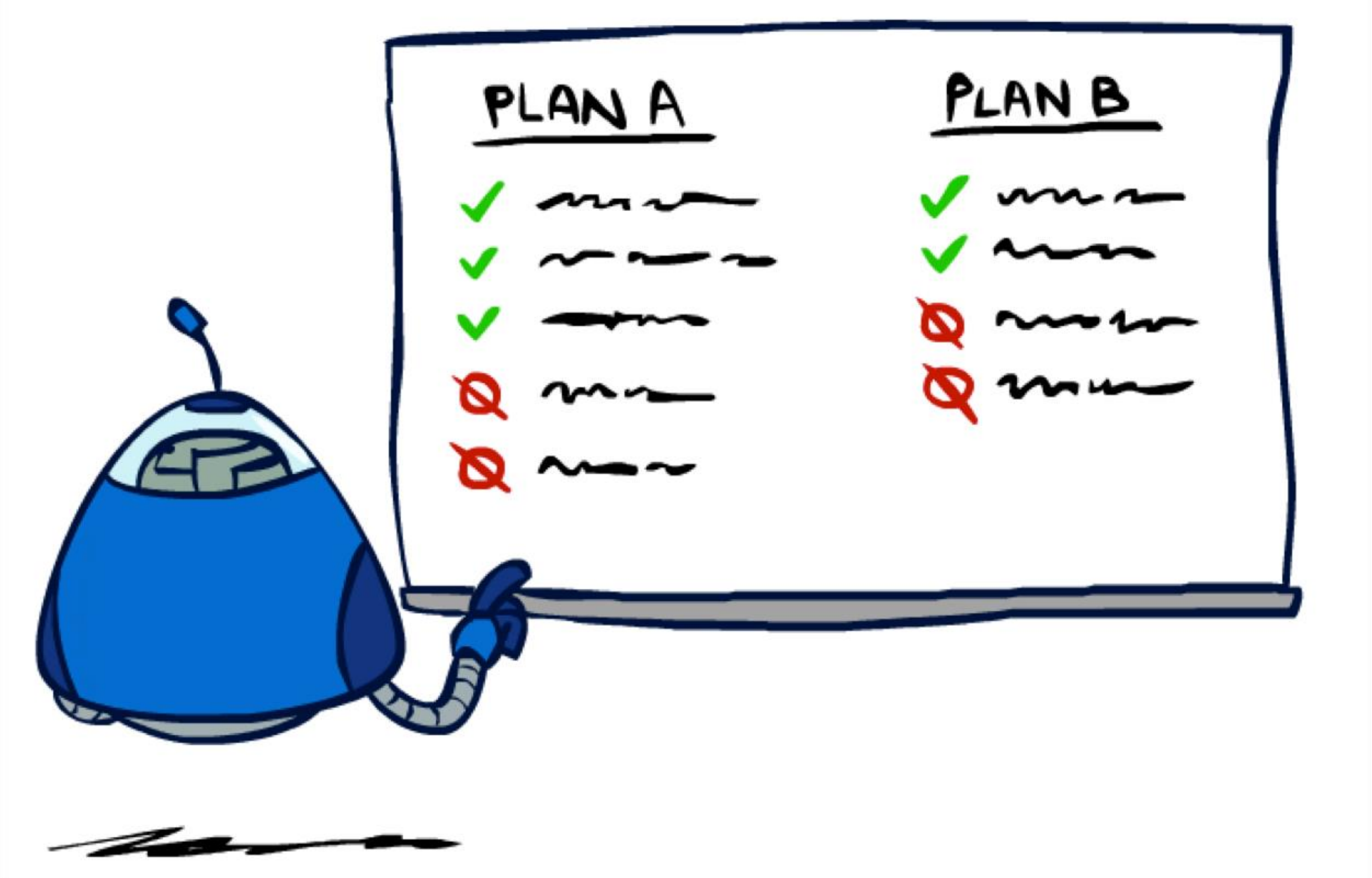
A Prize



A Lottery



Rationality

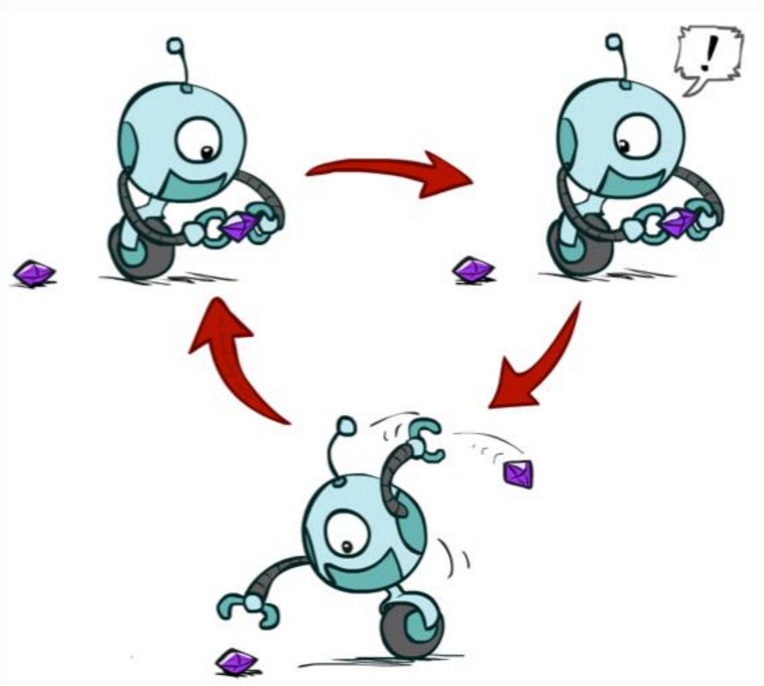


Rational Preferences

- We want some constraints on preferences before we call them rational, such as:

Axiom of Transitivity: $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

- For example: an agent with intransitive preferences can be induced to give away all of its money
  - If B > C, then an agent with C would pay (say) 1 cent to get B
  - If A > B, then an agent with B would pay (say) 1 cent to get A
  - If C > A, then an agent with A would pay (say) 1 cent to get C



Rational Preferences

The Axioms of Rationality

Orderability

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

Transitivity

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

Continuity

$$A \succ B \succ C \Rightarrow \exists p \; [p, A; \; 1-p, C] \sim B$$

Substitutability

$$A \sim B \Rightarrow [p, A; \; 1-p, C] \sim [p, B; 1-p, C]$$

Monotonicity

$$A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A; \; 1-p, B] \succeq [q, A; \; 1-q, B])$$

Theorem: Rational preferences imply behavior describable as maximization of expected utility

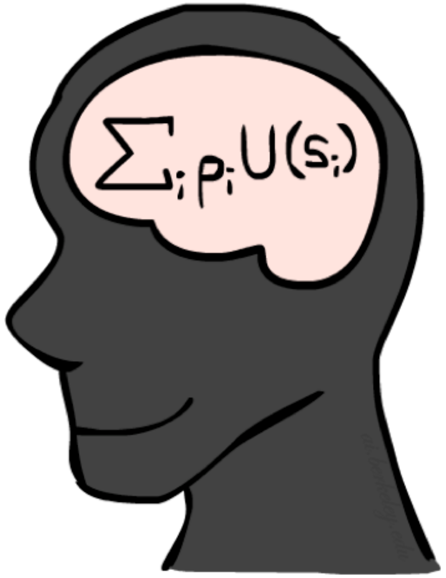
MEU Principle

- Theorem [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:

$$U(A) \geq U(B) \;\Leftrightarrow\; A \succeq B$$

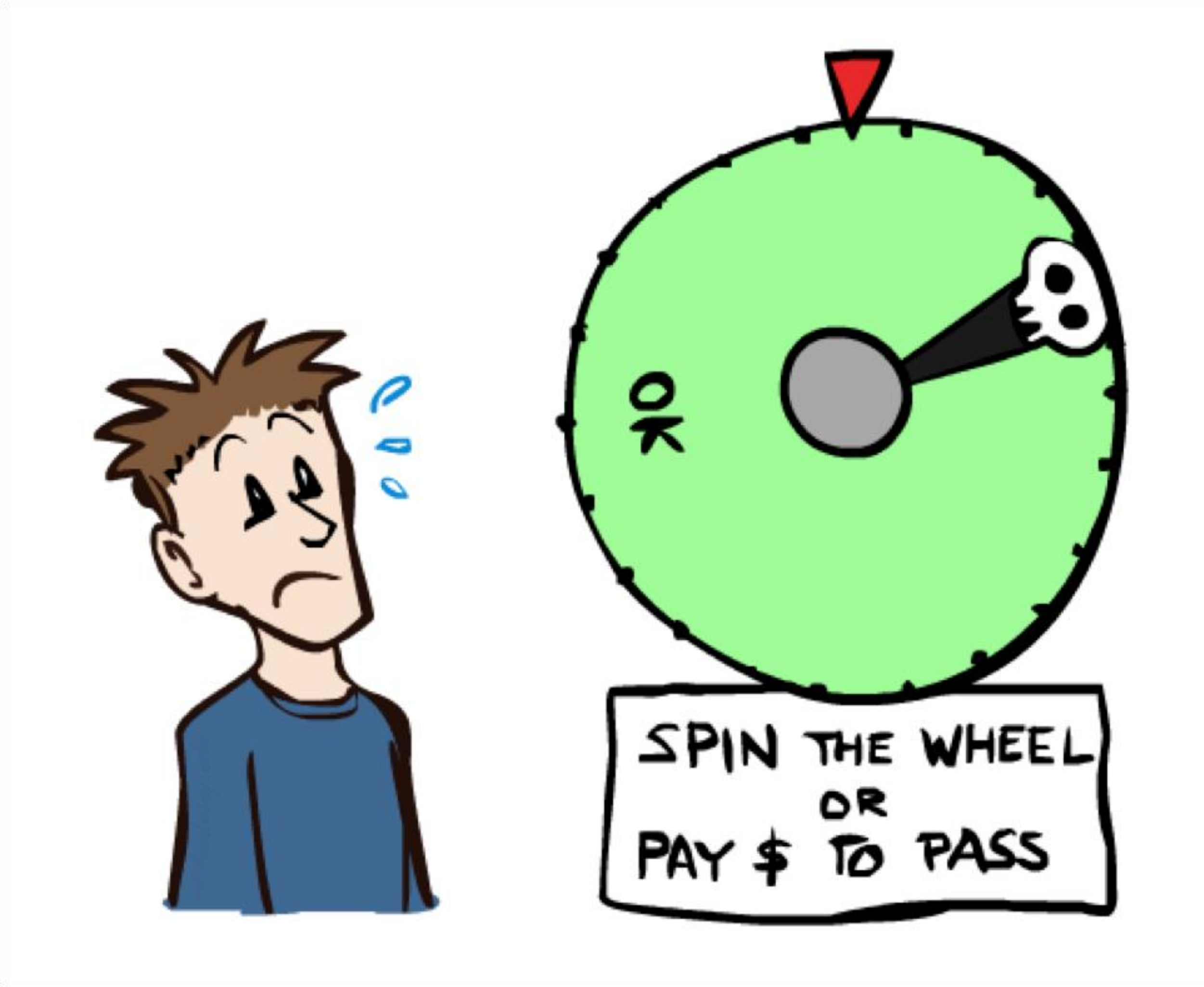
$$U([p_1, S_1;\; \ldots\; ;\; p_n, S_n]) = \sum_i p_i U(S_i)$$

  - I.e. values assigned by U preserve preferences of both prizes and lotteries!



- Maximum expected utility (MEU) principle:
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tic-tac-toe, a reflex vacuum cleaner

Human Utilities



Utility Scales

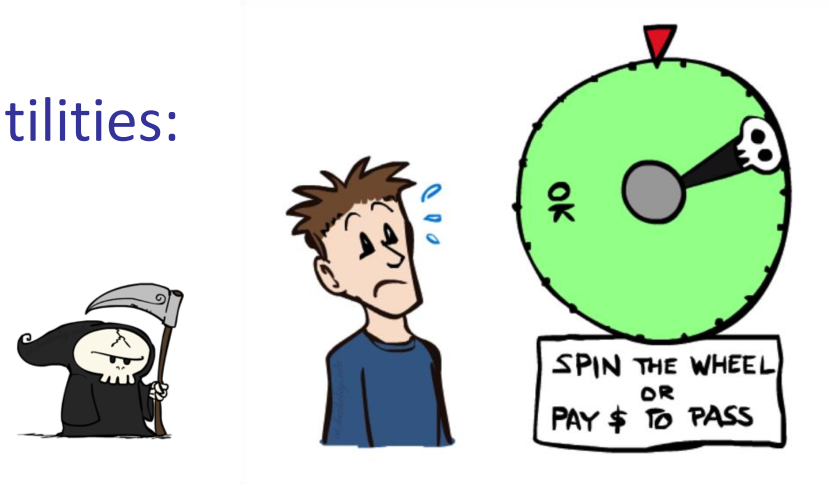
- Normalized utilities: $u_+ = 1.0$, $u_- = 0.0$
- Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.
- QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk
- Note: behavior is invariant under positive linear transformation

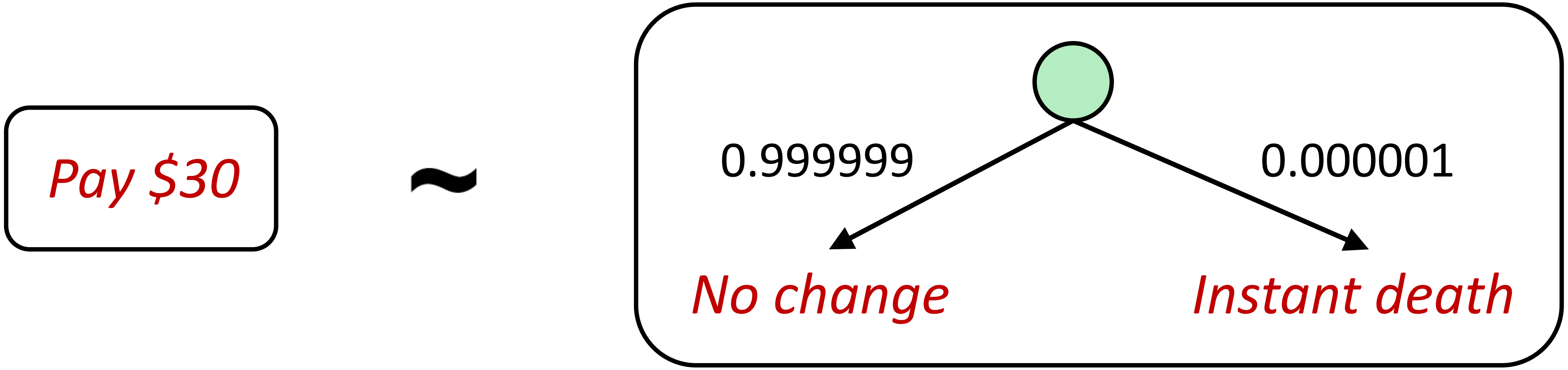
$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

- With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes

Human Utilities

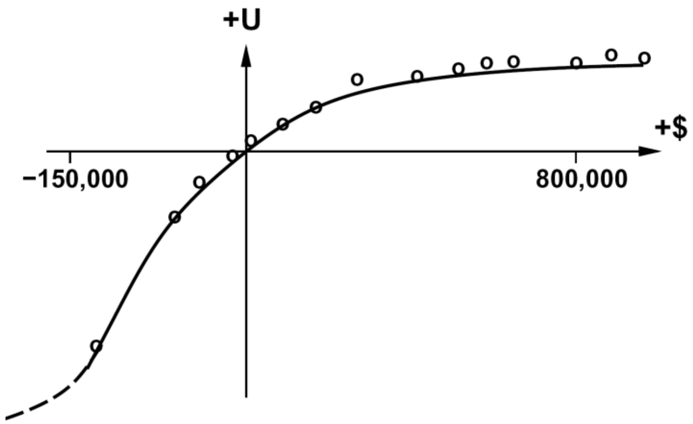
- Utilities map states to real numbers. Which numbers?
- Standard approach to assessment (elicitation) of human utilities:
  - Compare a prize A to a standard lottery L_p between
    - "best possible prize" u_+ with probability p
    - "worst possible catastrophe" u_- with probability 1-p
  - Adjust lottery probability p until indifference: $A \sim L_p$
  - Resulting p is a utility in [0,1]





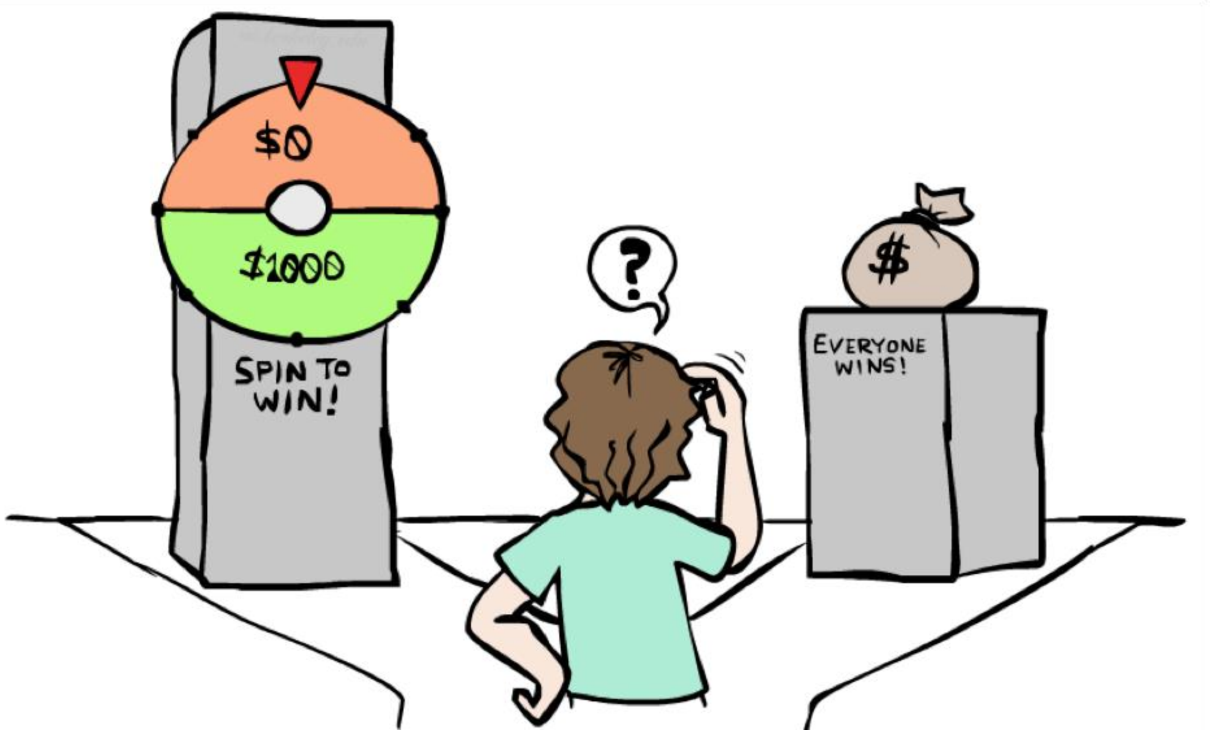
Money

- Money <u>does not</u> behave as a utility function, but we can talk about the utility of having money (or being in debt)
- Given a lottery L = [p, $X; (1-p), $Y]
  - The expected monetary value EMV(L) is p*X + (1-p)*Y
  - U(L) = p*U($X) + (1-p)*U($Y)
  - Typically, U(L) < U(EMV(L))
  - In this sense, people are risk-averse
  - When deep in debt, people are risk-prone



Example: Insurance

- Consider the lottery [0.5, $1000; 0.5, $0]
  - What is its expected monetary value? ($500)
  - What is its certainty equivalent?
    - Monetary value acceptable in lieu of lottery
    - $400 for most people
  - Difference of $100 is the insurance premium
    - There's an insurance industry because people will pay to reduce their risk
    - If everyone were risk-neutral, no insurance needed!
  - It's win-win: you'd rather have the $400 and the insurance company would rather have the lottery (their utility curve is flat and they have many lotteries)



Example: Human Rationality?

- Famous example of Allais (1953)
  - A: [0.8, $4k;   0.2, $0]
  - B: [1.0, $3k;   0.0, $0]
  - C: [0.2, $4k;   0.8, $0]
  - D: [0.25, $3k;   0.75, $0]
- Most people prefer B > A, C > D
- But if U($0) = 0, then
  - $B > A \Rightarrow U(\$3k) > 0.8\, U(\$4k)$
  - $C > D \Rightarrow 0.8\, U(\$4k) > U(\$3k)$

Today

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- Human Utilities